Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 1 Please read Chapter 1 of the Lecture Notes. This mission assumes previous experience with precalculus which is summarized in Chapter 1. The problems which follow intend to help make certain you have a solid understanding of the basics before we begin Calculus in Chapter 2.

Problem 2 Find A, B, C for which $\sqrt{\dfrac{8x^3\sqrt{xy^2}}{x^6\sqrt[3]{27y}}} = Cx^Ay^B$

Problem 3 Find A, B, C for which $\left[\dfrac{x}{\sqrt{36y}}(x^2y)\right]^{-1} = Cx^Ay^B$

Problem 4 Find A, B, C for which $\dfrac{x^2}{y^{-1}} \cdot \dfrac{\frac{y}{x}}{2/\sqrt{x/y}} = Cx^Ay^B$

Problem 5 Find A, B, C for which $\left[\dfrac{2x}{3y}\sqrt[3]{x^6y^3}\right]^2 = Cx^Ay^B$

Problem 6 Find A, B for which $\dfrac{3x}{x^2-16} = \dfrac{A}{x+4} + \dfrac{B}{x-4}$.

Problem 7 Suppose $(x+2y)^8 = x^8 + Ax^7y + \cdots + Bxy^7 + C$ then find the values of A, B, C.

Problem 8 If $f(x) = 2^x = e^{kx-1}$ for all $x \in \mathbb{R}$ then find k.

Problem 9 Suppose $x, y > 0$. Find A, B, C for which $3 + \log(x^3y) - 2\log(xy) = \log(Cx^Ay^B)$.

Problem 10 Consider the number systems $\mathbb{Z}, \mathbb{N}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$. Name each of these sets and explain how they can be arranged as subsets and supersets of one another.

Problem 11 Let $S = (-\infty, 3]$ and $T = (0, \infty)$. Write $S \cup T$ and $S \cap T$ as intervals of $\mathbb{R}$.

Problem 12 Let $A = [1, 4]$ and $B = \{1, 2, 3, 4\}$. Express $A - B$ in interval notation.

Problem 13 Let $S = \{(x, y) \mid 1 < x < 2,\ 3 \leq y \leq 4\}$. Express S as the Cartesian product of intervals of real numbers.

Problem 14 Let $S = \{x \in \mathbb{R} \mid |3x - 12| < 18\}$. Express S using interval notation and $\cup$ if necessary.

Problem 15 Let $S = \{x \in \mathbb{R} \mid 6 < |3x - 12|\}$. Express S using interval notation and $\cup$ if necessary.

Problem 16 Consider $U = \{x \in \mathbb{R} \mid -1 \leq x < 3\}$. Find any boundary points of U and determine the interior of U.

Problem 17 Find a center x_0 and radius ε for which $B_\varepsilon(x_0) = (3, 7)$.

Problem 18 Write $B_5(-3) \cap B_6(4)$ in interval notation.

Problem 19 For each function given below, find the natural domain and express the domain in interval notation.

(a.) $f(x) = \dfrac{x^2 - 2x}{x}$

(b.) $f(x) = \sqrt{3x - 7}$

(c.) $f(x) = \dfrac{3x - 2}{(x^2 + 4x + 5)^2}$

(d.) $f(x) = \ln(x - 8) + \log(x - 10)$

(e.) $f(x) = \sqrt[3]{x^2 - 4}$

Problem 20 Let $f(x) = \sqrt{5x + 4}$. Find $f[0, 1]$ and $f^{-1}[0, 1)$.

Problem 21 Let $f(x) = x^3 + 2$ and $A = \{1, 2, 3\}$ and $B = \{3, 10, 29, 123\}$. Find $f(A)$ and $f^{-1}(B)$.

Problem 22 Let $f(x) = \dfrac{3x-1}{x+2}$ find the formula for $f^{-1}(y)$ and find the domain and range of both f and f^{-1}.

Problem 23 Let $f(x) = x^2 - 2x + 5$ where $1 \leq x \leq 2$. Find the formula for $f^{-1}(x)$ and graph $y = f(x)$ and $y = f^{-1}(x)$.

Problem 24 Let $f(x) = 3 + 4e^{x-3}$. Find the formula for $f^{-1}(y)$ and find the domain and range of both f and f^{-1}.

Problem 25 Let $f(x) = \sin^{-1}(\pi x - 2)$. Find the formula for $f^{-1}(y)$ and find the domain and range of both f and f^{-1}.

Problem 26 Let $f(x) = \tan^{-1}(3x)$. Find the formula for $f^{-1}(y)$ and find the domain and range of both f and f^{-1}.

Problem 27 Let $f(x) = \sin^{-1}(x)$ and $g(x) = \csc(x) = \frac{1}{\sin x}$. They're both *inverses* for the sine function. Explain the difference between f and g.

Problem 28 Find the solution set of $\cos(2x) = \frac{1}{2}$.

Problem 29 Find the solution set of $\sin(\pi x) \geq 0$.

Problem 30 Suppose $\cot\theta = x$ where $x > 0$. Find algebraic expressions for $\tan\theta, \cos\theta, \sin\theta, \csc\theta$ and $\sec\theta$. Draw a triangle to guide your work.

Problem 31 Solve $\cosh(x) = 2$ and also solve $\cosh(x) = 0$ where $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.

Problem 32 Show $\cosh^2\phi - \sinh^2\phi = 1$.

Problem 33 Derive the formula for $\sinh^{-1}(x)$ as the natural log of an algebraic function.

Problem 34 Solve $\sinh^2(3x - 1) = 8$.

Problem 35 Factor each polynomial below completely over $\mathbb{C}$:

(a.) $f(x) = x^2 - 10x + 26$

(b.) $f(x) = 2x^2 + 10x - 3$

(c.) $f(x) = x^6 - 81x^2$

(d.) $f(x) = x^4 + 4x^2 + 1$

(e.) $f(x) = x^3 - 27$

Problem 36 Find a polynomial of least degree for which $p(1) = 0$, $p(-4) = 0$ and $p(-1 + i\sqrt{2}) = 0$ and $p(0) = 24$. Please leave your answer in factored form. Do not leave the polynomial in standard form.

Problem 37 Graph each rational function, be careful to label both vertical and horizontal asymptotes as well as x-intercepts.

(a.) $f(x) = \dfrac{3x-2}{(x+4)^2}$

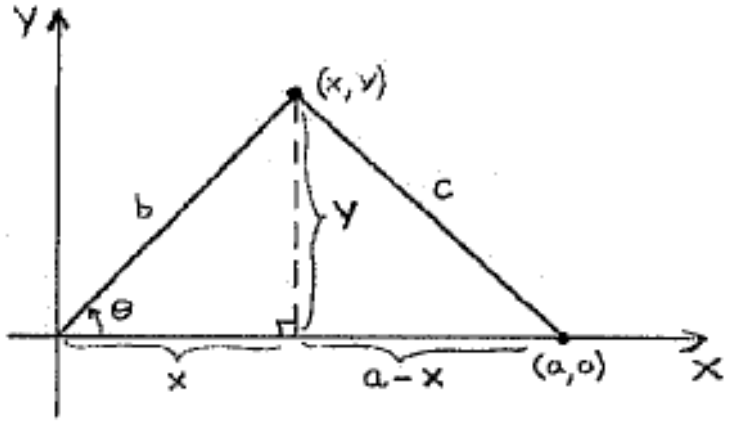
(b.) $f(x) = \dfrac{x^2-36}{4x^2-1}$

(c.) $f(x) = \dfrac{2x}{x^2-25}$

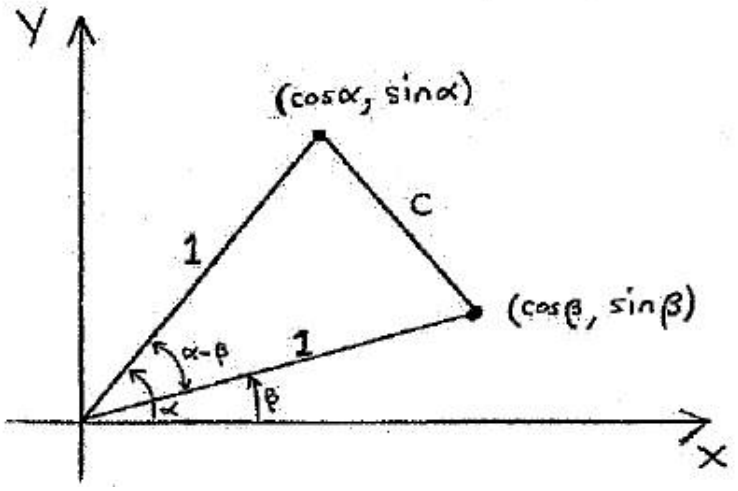
Problem 38 Let $f(x) = x^4 - 8x^2 + 8x + 15$. Notice $f(-1) = 0$ and $f(-3) = 0$. Factor $f(x)$ completely over $\mathbb{R}$.

Problem 39 Solve $x^4 - 8x^2 + 8x + 15 \geq 0$ and express the answer using interval notation.

Problem 40 Derive the law of cosines. In particular, for $a, b, c > 0$ as pictured below, show that $c^2 = a^2 + b^2 - 2ab\cos\theta$:



Problem 41 Consider the diagram below:



Apply the law of cosines in order to derive a formula for $\cos(\alpha - \beta)$. Then use the even/odd properties for sine and cosine to derive the adding angles formula $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

Problem 42 Does the identity $\cosh(A+B) = \cosh A \cosh B - \sinh A \sinh B$ hold true ? If not, how do you need to modify it to make it true ?