Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due. I wise course of study might be to go lightly on Problems 46-57 and complete the more of the later problems to begin.

Problem 43 Please read Sections 2.1 - 2.4 of the Lecture Notes.

Problem 44 Find the equation of a line from $(a, f(a))$ to $(b, f(b))$ assuming $a \neq b$.

Problem 45 Let $U = \{1, 2, 3\} \cup [4, 6) \cup (6, 7)$. Find all limit points of U. Which limit points of U are not interior limit points ? Which points are isolated points ?

Problem 46 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to 3}(2x + 7) = 13$.

Problem 47 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to -2}(8 - 2x) = 12$.

Problem 48 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to 3}|3x - 2| = 7$.

Problem 49 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to 3}(x^2 + 2x + 7) = 22$.

Problem 50 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to 2}(2x^2 - x - 3) = 3$.

Problem 51 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to -1}(x^3 - 10x^2 - 11x + 2) = 2$.

Problem 52 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to 7}\dfrac{2x}{2x + 7} = \dfrac{2}{3}$.

Problem 53 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to a}(x) = a$.

Problem 54 Prove from the $\varepsilon\delta$-definition of the limit that $\lim_{x\to a}\sqrt[3]{x} = \sqrt[3]{a}$.

Problem 55 Prove by the $\varepsilon\delta$-definition of the one-sided-limit that $\lim_{x\to 4^+}\sqrt{x - 4} = 0$.

Problem 56 Prove by the $\varepsilon\delta$-definition of the one-sided-limit that $\lim_{x\to 5^+}\left(3 + \sqrt{x - 5}\right) = 3$.

Problem 57 Prove by the $\varepsilon\delta$-definition of the limit that $\lim_{x\to 9}\left(3 + \sqrt{x - 5}\right) = 5$.

Problem 58 Calculate $\lim_{x\to 1}(13x^4 + 20x^2 + 9)$.

Problem 59 Calculate $\lim_{\theta\to \pi/4}\tan\theta$

Problem 60 Calculate $\lim_{\theta\to \pi^-}\tan\left(\dfrac{\theta}{2}\right)$

Problem 61 Calculate $\lim_{\theta\to \pi/4}\left[\sec\theta + \csc^2\theta\right]$

Problem 62 Calculate $\lim_{x\to 3^+}\sqrt{x - 3}$

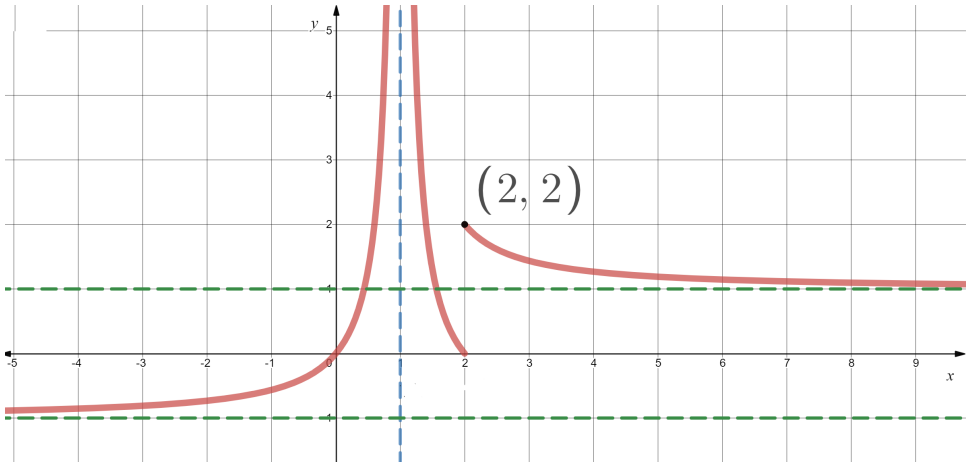
Problem 63 Calculate $\lim_{x\to -3}\left(\dfrac{x + 3}{x^2 - 9}\right)$

Problem 64 Calculate $\lim_{x\to 3^+}\left(\dfrac{x + 3}{x^2 - 9}\right)$

Problem 65 Given $\lim_{x\to 2} f(x) = 6$ calculate $\lim_{x\to 2}[7f(x)]$ and $\lim_{x\to 2}\cos(\pi f(x))$.

Problem 66 Let c be a constant and define $f(x) = \begin{cases} 3x-2 & x \leq 4 \\ 2x^3 + c & x > 4 \end{cases}$. Calculate both $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$. What value of c must we choose in order that $\lim_{x\to 4} f(x)$ exists.

Problem 67 See this website for this graph. Consider $y = f(x)$ given below:



Analyze the following limits: (worth 4pts)

(a.) $\lim_{x\to -\infty} f(x)$

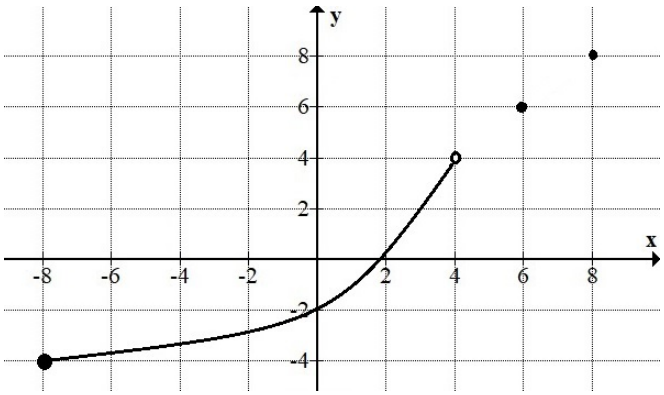
(b.) $\lim_{x\to \infty} f(x)$

(c.) $\lim_{x\to 1^-} f(x)$

(d.) $\lim_{x\to 1^+} f(x)$

(e.) $\lim_{x\to 1} f(x)$

Problem 68 Consider the graph $y = f(x)$ below:



(a.) which points are limit points of $f(x)$?

(b.) list the isolated points of $f(x)$,

(c.) find the limits at each left and right boundary point.

Problem 69 Suppose $\lim_{x\to 3^+} f(x) = 7$ and $\lim_{x\to 3^-} f(x) = 4$. Sketch the graph $y = f(x)$ near $x = 3$. What can you say about the two-sided limit at $x = 3$?

Problem 70 Suppose $\lim_{x\to 2^+} f(x) = 1$ and $\lim_{x\to 2^-} f(x) = 1$ yet $2 \notin dom(f)$. Sketch the graph $y = f(x)$ near $x = 2$. What can you say about the two-sided limit at $x = 2$?

Problem 71 Suppose $\lim_{x\to -3^+} f(x) = \infty$ and $\lim_{x\to -3^-} f(x) = -\infty$. Sketch the graph $y = f(x)$ near $x = -3$. What can you say about the two-sided limit at $x = 3$?

Problem 72 Prove $\lim\limits_{x\to 0} \dfrac{1}{x^2} = \infty$ by an argument directly from the definition of the divergent limit.

Problem 73 Sketch the graph of $y = \tanh x$ and determine $\lim_{x\to\infty} \tanh x$ as well as $\lim_{x\to -\infty} \tanh x$.

Problem 74 Determine $\lim\limits_{x\to \frac{\pi}{2}^+} \sec x$ as well as $\lim\limits_{x\to \frac{\pi}{2}^-} \sec x$ by examining the graph of cosine near $x = \pi/2$.

Problem 75 Calculate the infinite limits below using an algebraic technique: here the notation $0 < \delta << 1$ means that δ is an arbitrarily small positive quantity.

(a.) $\dfrac{x+3}{x-1}$ as $x \to 1^+$ (study $x = 1 + \delta$ where $0 < \delta << 1$)

(b.) $\dfrac{2x-9}{(x-4)^2}$ as $x \to 4$ (study $x = 4 \pm \delta$ where $0 < \delta << 1$)

(c.) $\dfrac{x+2}{x^2+x-12}$ as $x \to 3^-$ (study $x = 3 - \delta$ where $0 < \delta << 1$)

Problem 76 Prove the limit law: $\lim\limits_{x\to a}[f(x)+g(x)] = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$ given the limits of f and g both exist at $x = a$.

Problem 77 Prove the limit law: Suppose $c \in \mathbb{R}$ and $\lim\limits_{x\to a} f(x) = L \in \mathbb{R}$ then $\lim\limits_{x\to a} cf(x) = c \lim\limits_{x\to a} f(x)$.

Problem 78 Suppose $x^2 \leq f(x) \leq x^2 + 1$ for all x such that $0 < x < 2$. What can we say about $\lim_{x\to 1} f(x)$? Discuss.

Problem 79 Suppose $t^2 \leq g(t) \leq 3t$ for all t such that $0 \leq t < 1$. What can we say about $\lim_{t\to 0^+} g(t)$? Discuss.

Problem 80 Calculate the following limits via appropriate application of limit laws and algebra:

(a.) $\lim\limits_{x\to 1}(ax^2 + bx + c)$

(b.) $\lim\limits_{x\to a} \dfrac{3x+7}{x^2+4x+5}$

(c.) $\lim\limits_{x\to \ln 2} \cosh(4x)$

(d.) $\lim\limits_{x\to \pi} \cos^2(x)$

(e.) $\lim\limits_{x\to 0} \log(e^x + 9)$

(f.) $\lim\limits_{x\to 1} \ln(x^2 - 2x + 1)$

(g.) $\lim\limits_{x\to 5} \sqrt{x^3 - x + 1}$

(h.) $\lim\limits_{x\to 3^+} \dfrac{10}{x-3}$

(i.) $\lim\limits_{x\to 3^+} \dfrac{3x-9}{x^2-9}$

(j.) $\lim\limits_{t\to -2} \dfrac{t^3+8}{t+2}$

(k.) $\lim\limits_{y\to -8} \dfrac{y^2+9y+8}{2y^2-128}$

(l.) $\lim\limits_{x\to 2^-} \dfrac{x^4-16}{\sqrt{(x-2)^2}}$

(m.) $\lim\limits_{x\to 2^+} \dfrac{\sqrt{(2-x)^2}}{x-2}$

(n.) $\lim\limits_{x\to 4^+} \dfrac{3-x}{x^2-2x-8}$

(o.) $\lim_{x\to 4} \dfrac{4-x}{2-\sqrt{x}}$

(p.) $\lim_{z\to 6} \dfrac{z+6}{z^2-36}$

(q.) $\lim_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

(r.) $\lim_{x\to -2+\sqrt{3}} \dfrac{x+2-\sqrt{3}}{x^2+4x+1}$

Problem 81 Calculate the following limits via intuition guided by algebra where appropriate:

(a.) $\lim_{x\to\infty} \dfrac{126\sqrt[3]{x}+1}{\sqrt[3]{27x}}$

(b.) $\lim_{x\to\infty} (x^2+1)$

(c.) $\lim_{x\to-\infty} (x^3+x+1)$

(d.) $\lim_{x\to\infty} \tanh(2x)$

(e.) $\lim_{x\to\infty} \tan^{-1}(1-2x)$

(f.) $\lim_{x\to\infty} (3+e^{4-x})$

(g.) $\lim_{t\to-\infty} (3+2^t)$

(h.) $\lim_{x\to\infty} \dfrac{3x^2-x+2}{x^2+7}$

(i.) $\lim_{x\to-\infty} \dfrac{\sqrt{5x^2-2}}{x+3}$

(j.) $\lim_{x\to\infty} \dfrac{7-5x^5}{x+13}$

(k.) $\lim_{x\to\infty} \left(\sqrt{x^2+3}-x\right)$

Problem 82 Calculate $\lim_{x\to a} \dfrac{1}{(16x^2-1)^2}$. If needed, break into cases.

Problem 83 Calculate $\lim_{x\to 1}\left[(x-1)^2 \sin\left(\dfrac{1}{x-1}\right)\right]$ via the Squeeze Theorem.

Problem 84 Calculate $\lim_{x\to 1^+} \sec\left(\dfrac{\pi x}{2}\right)$

Problem 85 Calculate $\lim_{\theta\to\frac{\pi}{4}} [\sin(2\theta)+\tan(\theta)]$

Problem 86 Calculate $\lim_{x\to 10^+} \dfrac{x-10}{x^2-100}$

Problem 87 Calculate $\lim_{x\to -10^+} \dfrac{x-10}{x^2-100}$

Problem 88 (2pts) Consider $f(x) = \dfrac{x-3}{x^3+7x^2-8x}$. Calculate $\lim_{x\to a} f(x)$ for all a which give a finite limit. Also, determine which values for a give $\lim_{x\to a} f(x)$ which does not exist.

Problem 89 Calculate $\lim_{x\to 2}(x^2+1)e^{x-2}$

Problem 90 Calculate $\lim_{x\to 0} \sinh x$. The **hyperbolic sine** is defined by $\sinh x = \frac{1}{2}(e^x - e^{-x})$.

Problem 91 Calculate $\lim_{x\to \ln 2} \cosh x$. The **hyperbolic cosine** is defined by $\cosh x = \frac{1}{2}(e^x + e^{-x})$.

Problem 92 Calculate $\lim_{x\to 5} \dfrac{\sqrt{x+11}-4}{x-5}$

Problem 93 Calculate $\lim_{h\to 0} \left(\dfrac{h}{\sqrt{1+h}-1} \right)$.

Problem 94 Calculate $\lim_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t} \right)$.

Problem 95 Suppose $x^2 - 6x + 10 \leq f(x) \leq \cos(2\pi x) + \sin(\pi x)$ for all $x \in (2, 4)$. Use the Squeeze Theorem to calculate $\lim_{x\to 3} f(x)$.

Problem 96 Calculate $\lim_{x\to \pi} \cos(x + \sin x)$.

Problem 97 Find a value for a such that the limit $\lim_{x\to 2} \dfrac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists. Given that choice of a, calculate the limit.

Problem 98 Calculate $\lim_{x\to 2} \left(\dfrac{\frac{1}{x^2-4}}{\frac{1}{x-2}} \right)$.

Problem 99 Calculate $\lim_{x\to 0} \left(\dfrac{1 + \frac{84}{x^2}}{1 + \frac{2}{x^2}} \right)$.