Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 100 Please read Sections 2.4, 2.5, 3.1 and 3.2 of the Lecture Notes.

Problem 101 If f and g are continuous functions with $f(2) = 4$ and

$$\lim_{x\to 2} [3f(x) + g(x)] = 54$$

then find the value of $g(2)$.

Problem 102 Find a value for k for which $f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$ defines continuous function f on $\mathbb{R}$.

Problem 103 Find a value for A for which $f(x) = \begin{cases} Ax^2, & x \le 2 \\ 2x + A, & x > 2 \end{cases}$ defines continuous function f on $\mathbb{R}$.

Problem 104 Find a value for m for which $f(x) = \begin{cases} \tan(mx)/x, & x < 0 \\ 3x + 2m^2, & x \ge 0 \end{cases}$ is continuous at zero.

Problem 105 Use the definition of continuity (in terms of a limit) to explain why the following functions are discontinuous at $x = a$ and classify the type of discontinuity.

(a.) $a = 2$ for $f(x) = \begin{cases} \frac{3}{x-2}, & x \ne 2 \\ 5, & x = 2 \end{cases}$.

(b.) $a = 3$ for $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & x \ge 3 \\ 4, & x < 3 \end{cases}$.

Problem 106 Find a value(s) for c for which $f(x) = \begin{cases} x^2 - c^2, & x < 4 \\ cx + 11, & x \ge 4 \end{cases}$ is continuous at zero.

Problem 107 Use the intermediate value theorem to prove there exists a real number c between 1 and 2 for which $c^2 = 3$.

Problem 108 Use the intermediate value theorem (IVT) to prove there is at least one real root for the equation $2x-3 = \sqrt[3]{x}$. Then use a calculator to find an interval of length 0.1 that contains a root.

Problem 109 Suppose f is continuous on $[1, 5]$ and the only solutions of the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.

Problem 110 Explain why $x^4 - 4x + 1 = 0$ has at least one solution on the interval $[1, 2]$. Do not try to solve the equation. The point of this problem is for you to understand and apply a theorem covered in Chapter 2.

Problem 111 Suppose $a, b > 0$. Explain why $\dfrac{a}{x-1} + \dfrac{b}{x-3} = 0$ has at least one solution on the interval $(1, 3)$.

Problem 112 Find a value for c such that function below is continuous at $x = 2$. Explain your choice in terms of arguments with limits.

$$f(x) = \begin{cases} cx - 1 & if\ x \le 2 \\ \frac{4c(x-2)}{x^2-4} & if\ x > 2 \end{cases}$$

Problem 113 Let $f(x) = 10 + \dfrac{(x^2-4)(x^2-9)}{x^2-5x+6}$ for $x \ne 2, 3$. If $f(x) = g(x)$ for all $x \in \mathbb{R} - \{2, 3\}$ and g is a continuous function then find the values of $g(2)$ and $g(3)$. How are the graphs $y = f(x)$ and $y = g(x)$ related ?

Problem 114 Show f defined below is discontinuous at $x = 2$.

$$f(x) = \begin{cases} 3|x-2|(x-2)^{-1} & if\ x < 2 \\ 3 & if\ x \geq 2 \end{cases}$$

Problem 115 Suppose f is a continuous function such that $f(3) = \ln 2$. Calculate $\lim_{x\to 3} e^{3f(x)}$.

Problem 116 Use the IVT to prove that there exists a solution to the equation below: hint: think about the interval $[0, 4]$.

$$\sqrt{4x} - \sqrt[3]{x/4} = 1$$

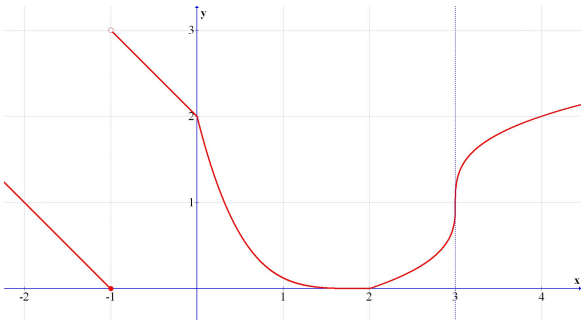
Problem 117 Apply the IVT to show there exists a solution of $e^x = x^4$ for some $x > 0$.

Problem 118 Let $f(x) = 3x^3 - 2x + x + 22$. Show there exists $c \in \mathbb{R}$ for which $f(c) = 0$.

Problem 119 Find the equation for the tangent line to $f(x) = x^3 + 2x$ at $(2, f(2))$. Your solution will include an explicit calculation of the slope of the tangent lines via limit-law type calculations.

Problem 120 Suppose the graph $y = f(x)$ has tangent line $y = 13(x-3) - 20$ with point of tangency $(3, -20)$. Find $f(3)$ and $f'(3)$.

Problem 121 Read the derivatives from the graph of $y = f(x)$ given below, if possible. If not possible briefly explain why.



$(a.) f'(-1)$ $(b.) f'(0)$ $(c.) f'(1.7)$ $(d.) f'(3)$

Problem 122 Find $f'(3)$ given $f(x) = 2x^2 + 7x - 3$

Problem 123 Suppose $a \neq 2$. Calculate $f'(a)$ for $f(x) = \dfrac{x}{3x-6}$.

Problem 124 Suppose $a \neq 3$. Calculate $f'(a)$ for $f(x) = \dfrac{2}{1+\sqrt{x-3}}$.

Problem 125 Let $f(x) = 5x^2 + 17x + 1$. Show $f'(-1) = 7$ by calculation from the limit definition of the derivative.

Problem 126 Let $f(x) = (x+3)^3 - x^2$. Calculate the value of $f'(0)$ directly from the limit definition of the derivative.

Problem 127 Let $f(x) = \sqrt[3]{x+1}$. Show $f'(1) = \frac{1}{3\sqrt[3]{4}}$ by calculation from the limit definition of the derivative.

Problem 128 Let $f(x) = \dfrac{1}{(2x-3)^2}$. Calculate the value of $f'(0)$ directly from the limit definition of the derivative.

Problem 129 Let $f(x) = 3x^4 - 2$. Calculate the value of $f'(a)$ directly from the limit definition of the derivative.

Problem 130 Let $f(x) = \dfrac{1}{\sqrt{x+3}}$. Calculate the value of $f'(1)$ directly from the limit definition of the derivative.

Problem 131 Suppose $a \neq 0$. Show from the definition of the derivative that $f'(a) = -1/a^2$ for $f(x) = 1/x$.

Problem 132 Suppose $a > 0$. Show from the definition of the derivative that $f'(a) = \frac{1}{2\sqrt{a}}$ for $f(x) = \sqrt{x}$.

Problem 133 Find the slope of the tangent line to $y = f(x)$ at $a = 2$ given $f(x) = x^2 + 3x - 1$. Also, find the equation of the tangent line to $y = f(x)$ at $x = 2$.

Problem 134 Suppose the position of a rock on mars at time t is given by $h = 60t - 3.72t^2$ where h is in meters and t in seconds.

(a.) find the velocity of the rock when $t = 2$,

(b.) find the velocity of the rock when $t = a$,

(c.) at what time will the rock hit the surface of mars which is at $h = 0$?,

(d.) at what velocity will the rock hit the surface of mars ?

Problem 135 Suppose the position of a ninja cat is given by $y(t) = 1 + 3t - 6t^2$. Find the velocity of the cat at time t_o.

Problem 136 Suppose the tangent line to $y = f(x)$ at $(-3, 2)$ passes through the point $(2, 1)$. Find $f'(-3)$.

Problem 137 Suppose the normal line to $y = f(x)$ at $(-3, 2)$ passes through the point $(8, 5)$. Find $f'(-3)$.

Problem 138 If the equation of the tangent line to $y = f(x)$ at $x = 2$ is given by $y = 3 + 42(x - 2)$ then find $f'(2)$ and $f(2)$.

Problem 139 Identify both the formula $f(x)$ and the point a for which:

(a.) $f'(a) = \lim_{h\to 0} \dfrac{\sqrt[3]{8+h} - 2}{h}$

(b.) $f'(a) = \lim_{h\to 0} \dfrac{1}{h}\left[\sin^2\left(\dfrac{\pi}{2} + h\right) - 1\right]$

(c.) $f'(a) = \lim_{x\to 3} \dfrac{1}{x-3}\left(2^x - 8\right)$

(d.) $f'(a) = \lim_{x\to \pi} \dfrac{\tan^3(x)\cos(3x)}{x - \pi}$

Problem 140 Suppose $f(x) = x^2 + x$ when $x \leq 3$ and $f(x) = 8x - 12$ when $x > 3$. Is f differentiable at $x = 3$?

Problem 141 Suppose $f(x) = x^2 + 8$ whenever $x \leq 1$ and $f(x) = 2x + 6$ whenever $x > 1$. Is f differentiable at $x = 1$?

Problem 142 What values must we choose for m and b such that $f(x) = \begin{cases} x^3, & x \leq 2 \\ mx + b & x > 2 \end{cases}$ is differentiable on $\mathbb{R}$.

Problem 143 Let $f(x) = x^3 - 3x$. Find the slope of the tangent line to $y = f(x)$ at $x = a$. How many points are there with $f'(a) = 0$?