Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 196 Please read Sections 3.11-3.13 and Chapter 4 of the Lecture Notes.

Problem 197 Our proof of the power rule $\frac{d}{dx}x^n = nx^{n-1}$ initially assumed $n \in \mathbb{N}$. Assume $x > 0$ and $n \in \mathbb{R}$ and prove the rule via logarithmic differentiation.

Problem 198 Differentiate the given expression:

(a.) $\frac{d}{dx}\left[\frac{3}{x} + \sqrt[4]{16x}\right]$

(b.) $\frac{d}{dx}\left[\frac{x^2 + \sqrt{3x}}{x}\right]$

(c.) $\frac{d}{dx}\left[4\sin x - e^x\right]$

(d.) $\frac{d}{dx}\left[\pi^3 + x^\pi\right]$

(e.) $\frac{d}{dx}\left[\sqrt{2\sqrt{3x}}\right]$

(f.) $\frac{d}{dx}\left[\frac{3x - 2/x^2}{\frac{1}{\sqrt{4x}}}\right]$

(g.) $\frac{d}{dx}\left[(x^2+3)^4\right]$

(h.) $\frac{d}{dx}\left[\frac{1}{x^2} + x^2\right]^2$

(i.) $\frac{d}{dx}\left[(e^{x/2})^2 + e^{-3x}[e^{4x} + e^{3x}]\right]$

(j.) $\frac{d}{dx}\left[x^2 + \left(\sqrt{x}\cos^2 x + \sqrt{x}\sin^2 x\right)^{42+\frac{1}{42}}\right]^2$

Problem 199 Differentiate the given expression: **Assume $a, b, c \in \mathbb{R}$ are constants in the problems below.**

(a.) $\frac{d}{dx}\left[2^x x^2 + 2\right]$

(b.) $\frac{d}{dx}\left[\frac{x^3}{6 - \sqrt{2x+7}}\right]$

(c.) $\frac{d}{dx}\left[x^2\sin(ax^b e^{-cx})\right]$

(d.) $\frac{d}{dx}\left[a^{x^b} + c\sin\pi x\right]$

(e.) $\frac{d}{d\theta}\left[\sin\big(a\,\tan(b + c\,\theta)\big)\right]$

Problem 200 Differentiate the given function.

(a.) $y = e^x \sin x$

(b.) $y = e^{\sin x}$

(c.) $y = e^{x\sin x}$

(d.) $y = \frac{1}{x^3}\exp\left(x^2 + 5x - 1\right)$

(e.) $y = \sqrt{x^2 + e^{3x}}$

(f.) $y = 3^{2\tan\sqrt{x}} + x\tan^{-1}(x)$

Problem 201 Calculate $\frac{dy}{dx}$ given that:

(a.) $2x^2 + 4x + 3y^2 - 10y = 0$

(b.) $\tan(xy) = x + y$

(c.) $x\ln(x + y) = y^2$

(d.) $4xe^y = y^2 e^{2x}$

(e.) $y^3\cos(x^2 + y^2) = x^4$.

Problem 202 Suppose $y^2 = x^3 + 1$. Calculate y'' and leave your answer in terms of x and y.

Problem 203 Differentiate the given function.

(a.) $f(x) = \ln(\sec x + \tan x)$

(b.) $f(x) = \tan^{-1}(1 + x^2)$

(c.) $f(x) = x\cot^{-1}(x) + \sqrt{e^x + 7}$

(d.) $f(x) = \sinh(x)\sinh^{-1}(x)$

(e.) $f(x) = \tanh^{-1}(\sqrt{x})$

(f.) $f(x) = \text{sech}^{-1}(2x^3)$

Problem 204 Use logarithmic differentiation to find the derivative of

(a.) $y = \dfrac{\sin^3 x(x^2+3)^5}{\sqrt{x}}$

(b.) $y = \dfrac{e^{-x}\cos^2(x)}{2^{3x}\sin(x^3)}$

(c.) $y = e^{x^2}(x^2+1)^2(x^3+1)^3(x^4+1)^4$

(d.) $y = \dfrac{e^{-x^2}\sqrt{x+3}}{3^x}$

(e.) $y = \dfrac{(x+1)^2 e^{\sqrt{x}}}{(x+2)^2\sin(x)}$

Problem 205 Differentiate $y = e^{x^2}\sinh^{-1}(x)\tan^{-1}(x) + \sqrt{(x+1)(x+2)^3}$.

Problem 206 Calculate $\frac{d}{dx}\left(\left[\frac{x+3}{e^x}\right]^{\cos(x)+7}\right)$.

Problem 207 Show $f(x) = \cosh^{-1}(x) - \ln(x + \sqrt{x^2 - 1})$ is identically zero for all $x \geq 1$.

Problem 208 Find all horizontal tangents to the curve $x^4 + y^4 = xy$. There are 3. Use some technology to check your answers here. Sketch the graph and include the horizontal tangent lines you located. The algebra can be done without computer assistance, but it's difficult.

Problem 209 Find the equation of the tangent line to $x^3 + y^3 + y^2 = x$ at $x = (1, -1)$.

Problem 210 Find the equation of the tangent line at (x_o, y_o) on the curve $x^2 + 4x + y^2 - 2y = 20$. You may need to treat a couple cases separately if $\frac{dy}{dx}$ does not exist due to a vertical tangent to the curve. In these cases we can fall back on intuitive analytic geometry to write the equation of the tangent line.

Problem 211 A train travels due east from the station with a speed of 100mph and a bus travels due north from the same station with a speed of $50mph$. If the bus and train travel with constant speed[1] and leave the station at the same starting time then how far apart are the bus and train after one hour and how fast is the distance between the bus and train increasing ?

[1] I am not accounting for the acceleration process, so, if you wish think of 50 and 100 as average speeds

Problem 212 A 10ft ladder is sliding down a vertical wall at a rate of $2ft/s$ at the moment the base of the ladder is $4ft$ away from the base of the wall. How fast is the base of the ladder sliding horizontally away from the wall ?

Problem 213 Suppose a hot air balloon is $1000ft$ away from Billy is rising vertically with a speed of $100ft/min$. If the baloon is $400ft$ above the ground then how fast is the angle of inclination above the horizontal increasing from Billy's perspective ?

Problem 214 We can describe the motion of a particle in the xy-plane using the polar coordinates r, θ which are defined by $x = r\cos\theta$ and $y = r\sin\theta$. Given that $\frac{dr}{dt} = 10$ and $\frac{d\theta}{dt} = 2$ find the speed the object. Note: the speed in two-dimensional motion is defined to be the magnitude of the velocity *vector* and the formula for speed is simply speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$.

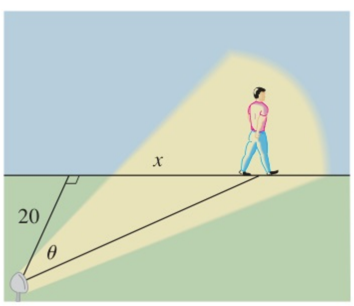
Problem 215 Let A, B, C be the lengths of the sides of a triangle with interior angle θ opposite C. If the area and angle θ of the triangle is held constant as the lengths of triangle are changed then what relation between A and B must hold as we deform the triangle ? How is the rate of change of C related in this situation ?

Problem 216 The volume of a sphere of radius R is given by $V = \frac{4}{3}\pi R^3$. If a spherical balloon with radius $R = 2.0ft$ is being inflated at a rate of $3ft^3/min$ then what is the rate at which the radius of the balloon increases ?

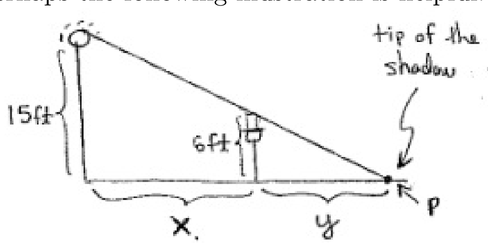
Problem 217 A circular oil slick has a radius which is increasing at the rate of 10 miles per day. At what rate is the area of the oil slick increasing if the radius of the slick is 3 miles ?

Problem 218 The volume of a cone of radius R with height h is given by $V = \frac{1}{3}\pi R^2 h$. If you have a belt which is $50cm$ wide and carries rocks at an average depth of $10cm$ and it moves rocks at a speed of $2m/s$ and drops them on the top of a large conical pile for which the radius is twice the height then find the rate at which the height of the pile increases given that $h = 10m$ at the time in question.

Problem 219 A man walks a straight path at a speed of $4ft/s$. A searchlight is located on the ground $20ft$ from the path and its beam is kept focused on the man. At what rate is the searchlight rotating when the man is $15ft$ from the point on the path closest to the searchlight[2] ?



Problem 220 A street light is mounted at the top of a $15ft$ pole. A man $6ft$ tall walks away from the pole with a speed of $5ft/s$ along a straight path. How fast is the tip of his shadow moving when he is $40ft$ from the pole ? Perhaps the following illustration is helpful:



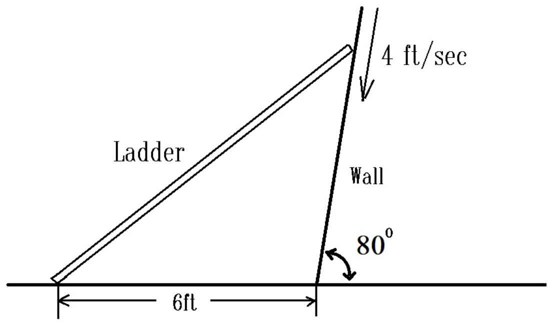
Problem 221 Newton's Law of cooling states that $\frac{dT}{dt} = k(T - R)$ where R is room temperature and T is the temperature of some given object. I usually think of T as the temperature of a cup of coffee. Find the rate of change of the temperature a cup of coffee given the coffee has $T = 120^oF$ and the room temperature is at $R = 80^oF$,

[2] credit to Stewart's Calculus where I found this problem

but the room temperature is dropping at a rate of 5 degrees per hour. Further, you're given that a previous experiment revealed $k = 10$ degrees per hour for the given insulated coffee cup. Additionally, find how fast the change in the temperature changing at the given time.

Problem 222 If a car travels around an ellipitical race track with equation $x^2 + 4y^2 = 200$ and its speed in the x-direction is decreasing at a rate of $10m/s$ when $x = 1$ then at what is the y-velocity ($\frac{dy}{dt}$)? There may be more than one reasonable answer.

Problem 223 If an extension ladder of length 20 ft leans against a wall which makes a 80^o angle with a horizontal yard and the top of the ladder is slides at a rate of 4 ft/s with respect to the wall then at what rate is the base of the ladder moving away from the base of the wall when the base of the ladder is 6 ft away from the base of the wall.



Problem 224 A person is standing 200 ft away from a model rocket that is fired straight up into the air at a constant rate of 10 ft/sec. At what rate is the distance between the person and the rocket increasing

(a.) 20 seconds after liftoff?

(b.) 1 minute after liftoff?

Problem 225 A go-cart travels over a hill which follows the graph $y = 4 - x^2$ for $-2 \leq x \leq 2$ in meters. If the go-cart has $\frac{dx}{dt} = 2\ m/s$ when $x = 1.5\,m$ then find $\frac{dy}{dt}$ at that instant in time.

Problem 226 Find the linearization L_f of $f(x) = \sqrt{x}$ based at $a = 4$. Calculate $L_f(4.1)$ and $L_f(9)$. Comment on the error in both approximations.

Problem 227 Find the linearization L_f of $f(x) = \sin x$ based at $a = 0$. Calculate $L_f(0.1)$ and $L_f(1)$. Comment on the error in both approximations.

Problem 228 Find the linearization L_f of $f(x) = x^3 + 3x + 1$ based at $a = 1$.

Problem 229 Find the linearization L_f of $f(x) = \sqrt[3]{1+x}$ based at $a = 0$. Calculate $L_f(0.1)$ and $L_f(1)$. Comment on the error in both approximations.

Problem 230 Find the linearization L_f of $f(x) = e^x$ based at $a = 0$ and calculate $L_f(1)$ and $L_f(-1)$. Comment on the error in both approximations. It would be good to draw a graph to explain your conclusions.

Problem 231 Solve the cubic equation $x^3 - 10x^2 + x + 2 = 0$ by using Newton's Method with an initial guess of $x_o = 8$ such that you are certain of the result to at least the hundreths place.

Problem 232 Solve $x = e^{-x}$ to with an error of not more than 0.01 by an application of Newton's Method to the function $f(x) = x - e^{-x}$. Begin the iteration with an initial guess of $x_o = 1$.