Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 233 Please read Sections 5.1 and 5.2 of the Lecture Notes.

Problem 234 Sketch a graph of a function which has:

(a.) a local maximum at 1 and is differentiable at 1

(b.) a local maximum on $[-2, 2]$ but no absolute maximum

(c.) has local maximum at 1 and is not continuous at 1.

Problem 235 Find all critical numbers for the following functions:

(a.) $f(x) = x^3 + 3x^2 - 24x + 4$

(b.) $g(x) = 16x^{1/3} - x^{4/3}$

(c.) $h(x) = |2x^2 + 5x - 18|$

(d.) $k(x) = 4x - \tan x$

Problem 236 Find critical numbers of $f(x) = \frac{1}{4}x^4 - 2x^3 + 6x^2 - 8x + 4$.

Problem 237 Find critical numbers of $f(x) = 2^{x^2-5x+6}$.

Problem 238 Find critical numbers of $f(x) = \tan^{-1}(e^x + x)$

Problem 239 Consider $f(x) = 5 - 12x + 3x^2$. Verify Rolles' Theorem for the given function on $[1, 3]$.

Problem 240 Consider $f(x) = x^3 - 3x - 1$. Verify the Mean Value Theorem for $f(x)$ on $[0, 2]$.

Problem 241 Find all critical values for the function $f(x) = \ln(x^2 + 2x + 7)$. Use the second derivative test to determine if the critical values determine a local minimum or maximum.

Problem 242 Let $f(x) = \tan x$. Show $f(0) = f(\pi) = 0$, but there does not exist $c \in (0, \pi)$ where $f'(c) = 0$. Why does this not contradict Rolle's Theorem ?

Problem 243 Show the equation $2x - 1 - \sin x = 0$ has exactly one real root.

Problem 244 Verify the function $f(x) = 3\sqrt{x+2}$ satisfies the hypothesis of the MVT on the interval $[-1, 2]$ and find all real numbers c that satisfy its conclusion.

Problem 245 Verify the function $f(x) = \frac{1}{12}\tan^2(x) + \cos x$ satisfies the hypothesis of the MVT on the interval $[-\pi/4, \pi/4]$ and find all real numbers c that satisfy its conclusion.

Problem 246 Let $f(x) = |x - 2|$. Show there is no c such that $f(3) - f(0) = f'(c)(3 - 0)$. Why does this not contradict the MVT ?

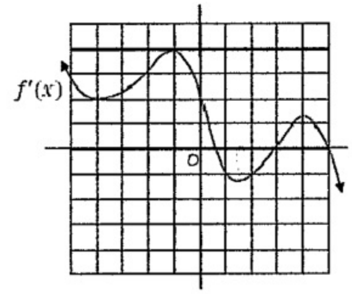
Problem 247 Consider $f(x) = \sqrt{(x-1)^2}$. Observe $f(0) = f(2) = 1$. Can you find $c \in [0, 2]$ such that $f'(c) = 0$? Does your result violate Rolles' Theorem? Discuss. Sketch the graph.

Problem 248 Consider $f(x) = \sqrt{(x-2)^2} - \sqrt{(x-3)^2}$. Observe $f(0) = -1$ and $f(4) = 1$. Can you find $c \in [0, 4]$ for which $f'(c) = \frac{f(4)-f(0)}{4} = \frac{1}{2}$? Is your result consistent with the claim of the Mean Value Theorem? Discuss. Sketch the graph.

Problem 249 If $f(3) = 4$ and $f'(x) \leq 5$ for $3 \leq x \leq 10$, how large can $f(10)$ possibly be ?

Problem 250 Suppose $2 \leq f'(x) \leq 5$ for all x. Show $4 \leq f(3) - f(1) \leq 10$.

Problem 251 The following is the graph of $\frac{df}{dx}$ with grid scale of 1-box per unit. Explain:



- **(a.)** On what intervals is f increasing ?
- **(b.)** At what values of x does f have a local maximum or minimum ?
- **(c.)** On what intervals is f concave upward or downward?
- **(d.)** What are the x-coordinates of the inflection points of f ?

Problem 252 Let $f(x) = x\sqrt{x+3}$.

- **(a.)** find all critical numbers
- **(b.)** find all intervals where f increasing or decreasing
- **(c.)** use first derivative test to find and classify all local extrema

Problem 253 Let $f(x) = x + \frac{4}{x}$.

- **(a.)** find all critical numbers
- **(b.)** find all intervals of concavity and inflection points
- **(c.)** use second derivative test to find and classify each local extrema.

Problem 254 Suppose $f(x) = x^2e^x$.

- **(a.)** find all critical numbers
- **(b.)** find all intervals of concavity and inflection points
- **(c.)** use second derivative test to find and classify each local extrema.

Problem 255 Suppose $f(x) = x - 2\sin x$ where $dom(f) = [0, 2\pi]$.

- **(a.)** find all critical numbers
- **(b.)** find all intervals of concavity and inflection points
- **(c.)** use second derivative test to find and classify each local extrema.

Problem 256 Let $f(x) = (x^2-3)e^x$ find local extrema and find intervals of increase and decrease. Include sign-chart for $f'(x)$ in your solution. Sketch the graph.

Problem 257 Let $f(x) = \exp(9+6x-x^2)$. Find critical numbers and determine the intervals of increase and decrease as well as the intervals of concavity. Classify any local extrema and find any inflection points. Include sign-charts for $f'(x)$ and $f''(x)$ for you solution. Sketch the graph.

Problem 258 Let $f(t) = \dfrac{t-1}{t^2-t+1}$. Find any critical numbers and classify the intervals of increase and decrease. Also classify any critical points. Include a sign-chart for $f'(x)$ in your solution. Sketch the graph.

Problem 259 Given $f(x) = x^3 + 3x^2 - 24x$. Find critical numbers and determine the intervals of increase and decrease as well as the intervals of concavity. Classify any local extrema and find any inflection points. Include sign-charts for $f'(x)$ and $f''(x)$ for you solution. Sketch the graph.

Problem 260 Consider $f(x) = (x+2)^2\ln(x^2+4x+5)$. Find all critical numbers for f. Find critical numbers and determine the intervals of increase and decrease. Classify any local extrema. Include sign-charts for $f'(x)$ and for your solution. Sketch the graph.

Problem 261 Consider $f(x) = \sin(x^2-3x+2)$ for $0 \leq x \leq 3$. Find critical numbers and determine the intervals of increase and decrease. Classify any local extrema. Include sign-charts for $f'(x)$ in your solution. Sketch the graph.

Problem 262 Consider $f(x) = x^2\sin(x)$ for $0 \leq x \leq 2\pi$. Find critical numbers and determine the intervals of increase and decrease as well as the intervals of concavity. Classify any local extrema and find any inflection points. Include sign-charts for $f'(x)$ and $f''(x)$ for your solution. Sketch the graph.

Problem 263 Find the absolute extrema of $f(x) = 3x^2 - 12x + 5$ on $[0, 3]$.

Problem 264 Find the absolute extrema of $f(x) = e^{\sin^2 x}$ on $[-\pi, \pi]$.

Problem 265 Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ which has a local maximum value of 3 at $x = -2$ and takes a local minimum value of 0 at $x = 1$.

Problem 266 Let $f(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$ where $a_n, \ldots, a_0$ are real constants and $a_n \neq 0$.

(a.) what is the maximum number of distinct zeros for this function ?

(b.) what is the maximum number of distinct critical points for this function ?

(c.) what is the maximum number of distinct inflection points for this function ?