

Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

**Problem 297** Please read Sections 6.1 - 6.4 of the Lecture Notes.

**Problem 298** Find the antiderivative  $F(x)$  for  $f(x) = 2x + 3$  for which  $F(0) = 1$ .

**Problem 299** If the velocity of an object as a function of time  $t$  is given by  $v(t) = 3e^t - 2t$  and the object is at  $x = 0$  when  $t = 0$  then find the position of the object as a function of time  $t$ .

**Problem 300** If the acceleration of an object as a function of time  $t$  given by  $a(t) = 6t^3$  and the initial velocity is  $v = 2$  whereas the initial position is  $x = 1$  when  $t = 0$  then find  $x(t)$  and  $v(t)$ .

**Problem 301** Find an antiderivative  $F(x)$  of  $f(x)$  for which  $F(0) = 0$ .

(a.)  $f(x) = 6x^2 - 8x + 3$

(b.)  $f(x) = 1 - x^3 + 12x^5$

(c.)  $f(x) = 5x^{1/4} - 7x^{3/4}$

(d.)  $f(x) = 2x + 3x^{1.7}$

(e.)  $f(x) = 10/x^9$

(f.)  $f(x) = \sqrt[3]{x^2} - \sqrt[3]{x^3}$

(g.)  $f(x) = \frac{x^3 + 2x^2}{\sqrt{x}}$

(h.)  $f(x) = 3 \cos x - 4 \sin x$

(i.)  $f(x) = \frac{3}{x^2} - \frac{5}{x^4}$

(j.)  $f(x) = 3e^x + 7 \sec^2(x)$

(k.)  $f(x) = 2x + 5(1 - x^2)^{-1/2}$

(l.)  $f(x) = \frac{x^2 + x + 1}{x}$

**Problem 302** Find the indefinite integrals indicated below.

(a.)  $\int x\sqrt{x} dx$

(b.)  $\int \frac{1 + 2x^2 + 3x^3}{\sqrt{x}} dx$

(c.)  $\int (3 + \cos x) dx$

(d.)  $\int \frac{dx}{3 + 3x^2}$

(e.)  $\int \frac{dx}{\sqrt{4 - 4x^2}}$

(f.)  $\int \tan^2 \theta d\theta$ : hint:  $\tan^2 \theta + 1 = \sec^2 \theta$

(g.)  $\int x(3 + x^4) dx$

(h.)  $\int [\cos^3(x) + \sin^2(x) \cos x] dx$

**Problem 303** Calculate the Left, Right and endpoint rules with  $n = 4$  ( calculate  $L_4, R_4$ ) to approximate the area bounded by  $y = f(x) = x^4 + x$  for  $0 \leq x \leq 4$ . Then, use the linked webtool: click on this link to billcookmath.com's sage-based area approximator-visualization tool to calculate  $L_{40}, R_{40}$ . Comment on whether  $L_n$  and  $R_n$  either over or under-estimate the true area bounded for  $y = f(x)$  on  $0 \leq x \leq 4$ .

**Problem 304** Calculate the Left, Right and endpoint rules with  $n = 4$  ( calculate  $L_4, R_4$ ) to approximate the area bounded by  $y = f(x) = \cos(3x/8)$  for  $0 \leq x \leq 4$ . Then, use the linked webtool: click on this link to billcookmath.com's sage-based area approximator-visualization tool to calculate  $L_{40}, R_{40}$ . Comment on whether  $L_n$  and  $R_n$  either over or under-estimate the true area bounded for  $y = f(x)$  on  $0 \leq x \leq 4$ .

**Problem 305** Observe that  $\frac{d}{dx} \frac{8}{3} \sin(3x/8) = \cos(3x/8)$ . This shows you an antiderivative for  $\cos(3x/8)$ . Apply FTC II to calculate  $\int_0^4 \cos(3x/8) dx$  and comment how your result compares to the numerical method used in previous problem.

**Problem 306** Rewrite the infinite sums below as a definite integral and then use the FTC II to find the value of the sum.

(a.)  $\lim_{n \rightarrow \infty} \sum_{j=1}^n \left(1 + \frac{2j}{n}\right)^3 \frac{2}{n}$

(b.)  $\lim_{n \rightarrow \infty} \sum_{j=1}^n \cos\left(\frac{\pi j}{2n}\right) \frac{1}{n}$

(c.)  $\lim_{n \rightarrow \infty} \sum_{j=1}^n \exp\left(\frac{j \ln(3/2)}{n}\right) \frac{\ln(3/2)}{n}$

**Problem 307** Use the FTC II to calculate the area bounded by  $y = x^4 + x$  with  $0 \leq x \leq 4$ . That is, calculate  $\int_0^4 (x^4 + x) dx$  via the amazing result of FTC II. Also, comment how your result compares to the numerical method used in Problem 138.

**Problem 308** Calculate  $\int_0^1 (x^3 + x^4 + x^5) dx$ .

**Problem 309** Calculate  $\int_0^{2\pi} \sin \theta d\theta$ . Does the result make sense graphically ?

**Problem 310** If  $\int_0^{\ln a} e^x dx = \int_0^{\pi/4} \sqrt{2} \cos x dx$  then find the value of  $a$ .

**Problem 311** Calculate  $\frac{d}{dx} \int_1^x \cosh^{-1}(t^2 + 3) dt$

**Problem 312** Calculate  $\frac{d}{dx} \int_{x^2}^{e^x} \frac{dt}{\sqrt{3+t}}$

**Problem 313** Calculate  $\int_{-1}^1 (t^2 + t^3) dt$

**Problem 314** Calculate  $\int_0^1 (\sqrt{x} + \sqrt[3]{x}) dx$

**Problem 315** Calculate  $\int_0^{\ln(2)} \cosh(x) dx$  and leave your answer as an exact fraction.

**Problem 316** Calculate  $\int_{-3}^{-1} \left(\frac{x+1}{x}\right) dx$

**Problem 317** Calculate  $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$

**Problem 318** Calculate  $\int_0^1 \frac{x^2}{1+x^2} dx$

**Problem 319** Calculate  $\int_0^\pi \cos(x/2) dx$

**Problem 320** Calculate  $\int_0^1 \sqrt{3x} dx$

**Problem 321** Calculate the following definite integrals<sup>1</sup>:

(a.)  $\int_{-1}^3 x^5 dx$

(b.)  $\int_1^2 x^{-2} dx$

(c.)  $\int_2^8 (4x+3) dx$

(d.)  $\int_0^4 (1+3y-y^2) dy$

(e.)  $\int_0^4 \sqrt{x} dx$

(f.)  $\int_\pi^{2\pi} \cos \theta d\theta$

(g.)  $\int_{-1}^0 (2x - e^x) dx$

(h.)  $\int_{-1}^0 (2x - e^x) dx$

(i.)  $\int_0^1 x^{3/7} dx$

(j.)  $\int_1^2 \frac{3}{\eta^4} d\eta$

(k.)  $\int_1^4 \frac{dx}{\sqrt{x}}$

(l.)  $\int_1^2 \frac{x^2+1}{\sqrt{x}} dx$

(m.)  $\int_0^2 (x^3-1)^2 dx$

(n.)  $\int_{\pi/4}^{\pi/3} \sin t dt$

(o.)  $\int_1^2 \frac{4+w^2}{w^3} dw$

(p.)  $\int_0^1 u(\sqrt{u} + \sqrt[3]{u}) dw$

(q.)  $\int_0^5 (2e^x + 4 \cos x) dw$

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<sup>1</sup>solutions to all of these are given in the pdf <http://www.supermath.info/integrationhwk.pdf> posted on my website

$$\text{(r.) } \int_{\pi/6}^{\pi/3} \csc^2 \theta \, d\theta$$

$$\text{(s.) } \int_1^8 \frac{x-1}{\sqrt[3]{x^2}} \, dx$$

$$\text{(t.) } \int_1^9 \frac{dx}{2x}$$

$$\text{(u.) } \int_{\ln 3}^{\ln 6} 8e^x \, dx$$

$$\text{(v.) } \int_8^9 2^t \, dt$$

$$\text{(w.) } \int_{\pi/3}^{\pi/2} \csc x \cot x \, dx$$

$$\text{(x.) } \int_0^{0.5} \frac{dx}{\sqrt{1-x^2}}$$

$$\text{(y.) } \int_0^{\pi/4} \frac{1 + \cos^2 \beta}{\cos^2 \beta} \, d\beta$$

$$\text{(z.) } \int_1^2 |x - x^2| \, dx$$

**Problem 322** Integration can be used to formulate a given function. Consider  $f(x) = \int_0^x \frac{dt}{1+t+t^2}$ . Find all zeros of  $f(x)$ . Determine the intervals of increase and decrease for the given function. Classify any local extrema. Find the interval(s) on which this function is concave up or concave down. Does  $y = f(x)$  have an inflection point ?