No graphing calculators and show your work with proper notation. There are at least 280pts to earn. Box your answers and work problems in the white space provided. Thanks and Enjoy!

1. Let $F(x) = \mid x^2 - 3x + 2 \mid$ be the force acting on some particle. Calculate the work done by this force over the interval $[0, 3]$.

2. Calculate $\int \sin^3 \theta \; d\theta$.

3. Integrate
$$\int \frac{x+2}{x^2+4x+3}\, dx$$

4. Calculate the improper integral below. Show the limits as we did in lecture. If an indeterminant limit arises in the calculation show how it is determined.

$$\int_0^1 \ln(3x)\, dx.$$

5. Integrate.

$$\int_0^\infty xe^{-x}.$$

6. Integrate.

$$\int \sqrt{16 - x^2}\, dx$$

7. Set-up, but do not explicitly calculate, the partial fractions decomposition for:

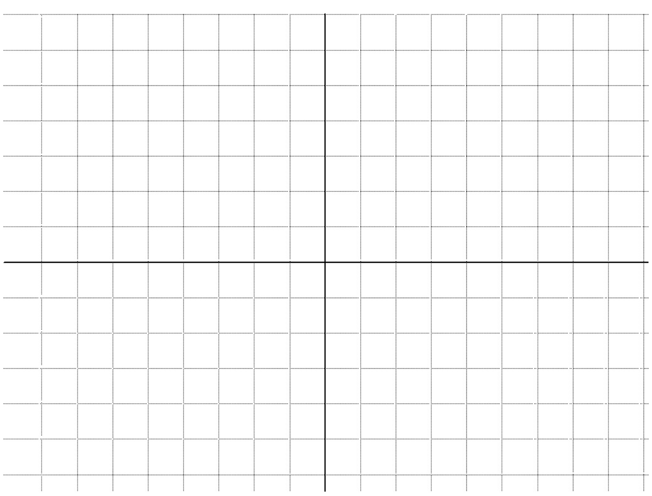
$$\frac{x^2+3x+2}{x(x-3)^3(x^2+1)}$$

8. Solve $\frac{dy}{dx} = e^{x-y}$.

9. Solve

$$\frac{dy}{dx} + y = 3^x$$

10. Suppose $x(t) = 3 + 2\cos(t)$ and $y(t) = \sin(t) - 3$ for $0 \le t \le \pi$ parametrizes a curve $C$. Determine the Cartesian equation of $C$ including inequality(s) if necessary. Sketch $C$ on the plot provided and find the equation of the tangent line at $t = \pi/4$.



11. Let $\vec{v}(t) = \left\langle e^{2t},\ 3t^2 \right\rangle$ denote the velocity of some particle at time $t$. You are given that the particle is at $(2, 1)$ at time zero. Derive the following:

    (a) the acceleration as a function of time $t$

    (b) the position as a function of time $t$

12. Find the Cartesian form of the polar equation $r = 2\cos(\theta)$ and identify the curve.

13. Give the parametric equations for the line segment that goes from $(4, 3)$ to $(1, 2)$. Set-up your parametrization so that $t = 0$ gives the point $(4, 3)$ and $t = 1$ gives the point $(1, 2)$.

14. Consider $\{\ln(n) - \ln(n+1)\}_{n=1}^{\infty}$. List the first 3 elements in the sequence. Does this sequence converge or diverge?

15. Suppose $\{a_n\}_{n=1}^{\infty}$ where $a_1 = 1$ and $a_{n+1} = 3 - \frac{1}{a_n}$ for all $n \in \mathbb{N}$. List the first 3 elements in the sequence. Next elaborate on how this sequence is bounded and find its limit if it does indeed converge.

16. Does $\sum_{n=1}^{\infty} \frac{3n^2+1}{n^3+n+42}$ converge or diverge? Support your answer with appropriate tests and complete arguments.

17. Does $\sum_{n=0}^{\infty} \frac{7^n}{n!}$ converge or diverge? Support your answer with appropriate tests and complete arguments.

18. Calculate the sum of the series $S = a_0 + a_1 + a_2 + \cdots$ given that

$$\sum_{j=0}^{n} a_j = 3 - \tan^{-1}(n)$$

for $n = 1, 2, 3, \ldots$. Also, find the formula for $a_j$ for $j = 0, 1, 2, \ldots$.

19. Find the IOC and ROC for:

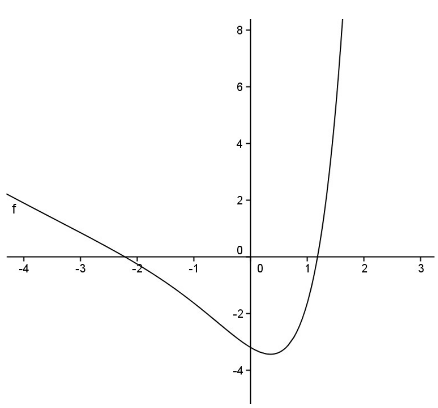
$$f(x) = \sum_{n=0}^{\infty} (3x - 2)^n$$

20. Calculate $f^{(99)}(0)$ for $f(x) = (1 + x^4)\sin(x^3)$

21. Find the first two nontrivial terms in the power series solution of the definite integral below and find a bound on the error in truncating the series to these two terms.

$$\int_0^{0.1} \frac{dx}{1 + x^7}$$

22. Suppose the function $f(x) = a_o + a_1(x+3) + a_2(x+3)^2 + a_3(x+3)^3 + \cdots$ has the graph below. What does the graph tell you about the **signs** of $a_o, a_1$ and $a_2$?



23. Calculate the power series expansion for $f(x) = \tan^{-1}(x)$ centered at $x_o = 0$. In addition, find the IOC for the series expansion you derive (you should include brief, but pointed arguments to explain why your IOC is justified).

24. Sketch the statement of Talyor's Theorem with Lagrange's form of the remainder. What important theorem did we use in the proof of Taylor's Theorem?

25. Suppose $p \in \mathbb{Z}$. Calculate:

$$\int_{-1}^{1} \sin(px) \ln(1 + x^{2p}) \; dx.$$

**you may choose ONE of the following pair to work**

26. Solve $y'' + 4y' + 5y = t + 2e^t$. The form of the particular solution is $y_p = At + B + Ce^t$ for this problem.

27. Calculate $\int x \sin(3x)\cos(8x)dx$

**you may choose ONE of the following pair to work**

28. Solve $y'' + 4y' + 4y = 0$ subject to the initial conditions $y(0) = 1$ and $y'(0) = 0$.

29. Write $f(x) = x^3$ as a power series centered at $x_o = 1$.

30. Suppose $C$ is a path which passes through $(0,0)$ at time zero and has $\frac{dy}{dt} = e^t$ and $\frac{dx}{dt} = t^4 - 3t^3 - 5t^2 + 29t - 30$. Furthermore, notice that $f(x) = x^4 - 3x^3 - 5x^2 + 29x - 30$ has a complex zero of $2+i$. Find all points where $C$ has vertical tangents.

31. Let $a, b$ be fixed positive constants. Find the surface area of the solid of revolution formed via rotating $x = at$ and $y = 1 + bt$ for $0 \leq t \leq 1$ around the $x$-axis. You should include a sketch which helps motivate your integral.