HOMEWORK 10 : §10.1 # 7 : CALCULUS II : (STEWART 6^{th} Ed.)

§10.1#7

$\boxed{\text{(a.) What can we say about a sol}^{\underline{n}} \text{ of } \frac{dy}{dx} = -y^2 \text{ just from looking at the DEq}^{\underline{n}}\text{?}}$

Notice $y^2 \geq 0$ thus $-y^2 \leq 0$ and so we find either $y = 0$ for all x or $y \neq 0$ but y is decreasing.

$\boxed{\text{(b.) Verify that all members of } y = \frac{1}{x+c} \text{ are sol}^{\underline{n}}\text{'s of } y' = -y^2.}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x+c}\right) = \frac{-1}{(x+c)^2}\frac{d}{dx}(x+c) = \frac{-1}{(x+c)^2} = -y^2.$$

Thus, $y = 1/(x+c)$ is a sol$^{\underline{n}}$ of $y' = -y^2$.

$\boxed{\text{(c.) Is there a sol}^{\underline{n}} \text{ of } y' = -y^2 \text{ that is not in the family of curves given in (b)?}}$

YES. It's not hard to guess $y = 0$ is a sol$^{\underline{n}}$. Moreover, once the guess is made it's easy to verify that $y' = -y^2$ if $y \equiv 0$.

identically zero. (it's zero for all x)

Notice

$$\frac{dy}{dx} = -y^2$$

$$\Rightarrow \underbrace{\frac{dy}{-y^2} = dx}_{} \Rightarrow \frac{1}{y} = x + c \Rightarrow y = \frac{1}{x+c}$$

this step assumes $y \neq 0$, we lose the $y = 0$ sol$^{\underline{n}}$ in the sep. of variables calculation. Sol$^{\underline{n}}$'s like this are called "exceptional"

$\boxed{\text{(d.) Find sol}^{\underline{n}} \text{ of } y' = -y^2 \text{ such that } y(0) = 0.5.}$

$$0.5 = \frac{1}{0+c} \Rightarrow \frac{1}{2} = \frac{1}{c} \Rightarrow \underline{c = 2}. \Rightarrow \underline{y = \frac{1}{x+2}}.$$