§12.2#10

a.) $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_{49} = \sum_{j=1}^{n} a_j$ (no difference except the lable.

b.) $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$

whereas $\sum_{i=1}^{n} a_j = \underbrace{a_j + a_j + \cdots + a_j}_{n\text{-times}} = n a_j$

different clearly.

§12.2#15

$$\sum_{n=1}^{\infty} 6(0.9)^{n-1} = \frac{a}{1-r} = \frac{6}{1-0.9} = \boxed{60}$$

by geometric series result with $a = 6$ and $r = 0.9$.

§12.2#31

$$\sum_{n=1}^{\infty} \underbrace{\arctan(n)}_{\tan^{-1}(n)}, \quad \text{notice } \lim_{n\to\infty} (\tan^{-1}(x)) = \frac{\pi}{2} \neq 0$$

$\therefore$ By n^{th} term test this series diverges.

§12.2#34

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}, \quad \text{notice } \lim_{n\to\infty} \frac{e^n}{n^2} \underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{n\to\infty} \left(\frac{e^n}{2n}\right) \underset{(\frac{\infty}{\infty})}{\overset{L}{=}} \lim_{n\to\infty} \left(\frac{e^n}{2}\right) = \infty \neq 0$$

(extending n to be real, continuous, variable)

$\therefore$ By n^{th} term test this series diverges.

§12.2#42

$$\begin{aligned}
0.\overline{73} &= 0.737373\ldots \\
&= 0.73 + \tfrac{1}{100}(0.73) + \left(\tfrac{1}{100}\right)^2(0.73) + \cdots \\
&= \frac{0.73}{1-\frac{1}{100}} \quad \text{geom. with } a = 0.73 \text{ \& } r = \tfrac{1}{100} \\
&= \frac{73/100}{99/100} = \boxed{\frac{73}{99}}
\end{aligned}$$

§12.2 #48

$$\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \sum_{n=1}^{\infty} \frac{x}{3}\left(\frac{x}{3}\right)^{n-1}$$

$$= \frac{x/3}{1 - x/3}$$ for $\left|\frac{x}{3}\right| < 1$ by geom. series result applied to $a = x/3$ and $r = x/3$

$$= \boxed{\frac{x}{3-x}}$$ ← for $|x| < 3$ or in other words $-3 < x < 3$.

§12.2 #56

Given $\sum_{n=1}^{\infty} a_n$ has n^{th} partial sum $S_n = 3 - n2^{-n}$

we find $$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty}\left(3 - n2^{-n}\right) = 3 - \underbrace{\lim_{n\to\infty}\left(\frac{n}{2^n}\right)}_{\text{goes to zero by L'Hopital's rule applied to this function real extension.}} = \boxed{3}$$

To find a_n notice that

$$S_n = a_n + S_{n-1}$$

$$\therefore \quad a_n = S_n - S_{n-1}$$

$$\Rightarrow a_n = \left(3 - n2^{-n}\right) - \left(3 - (n-1)2^{-(n-1)}\right)$$

$$\Rightarrow \boxed{a_n = (n-1)2^{1-n} - n2^{-n}}$$

§12.2 #60 Find c such that $\sum_{n=0}^{\infty} e^{nc} = 10$.

$$\sum_{n=0}^{\infty} e^{nc} = 1 + e^{nc} + \left(e^{nc}\right)^2 + \cdots$$ is geom. with $a = 1$, $r = e^{c}$

We want $$10 = \frac{1}{1 - e^{c}}$$

$$10 - 10e^{c} = 1 \longrightarrow e^{c} = \frac{9}{10}$$

$$\longrightarrow \boxed{c = \ln(9/10)}$$