§12.4 #3 Conv / Div ?

$$S = \sum_{n=1}^{\infty} \frac{n}{2n^3+1} \quad , \text{ has positive terms, can use D.C.T.}$$

Notice $\frac{n}{2n^3+1} \leq \frac{n}{2n^3} = \frac{1}{2}\left(\frac{1}{n^2}\right)$.

Also $\sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{1}{n^2}\right) = \frac{1}{2}\left(\sum_{n=1}^{\infty} \frac{1}{n^2}\right)$ which converges by $p=2$ series test. Thus by D.C.T. we find S converges.

§12.4 #5 Conv / Div ?

$$S = \sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}} \quad , \text{ has positive terms can apply D.C.T.}$$

Notice $\frac{n+1}{n\sqrt{n}} \geq \frac{n}{n\sqrt{n}} = \frac{1}{\sqrt{n}}$

But $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by $p = \frac{1}{2}$ series test.

Hence by D.C.T. S diverges.

§12.4 #8 Conv / Div ?

$$S = \sum_{n=1}^{\infty} \frac{4+3^n}{2^n} = \sum_{n=1}^{\infty} \left(\frac{4}{2^n} + \left(\frac{3}{2}\right)^n\right)$$

oops! notice $\frac{4}{2^n} + \left(\frac{3}{2}\right)^n \longrightarrow \infty$ as $n \to \infty$

$\therefore$ S diverges by n^{th} test.

§12.4 #12 Conv / Div.

$$S = \sum_{n=0}^{\infty} \frac{1+\sin(n)}{10^n} \qquad \text{notice } 0 \leq \frac{1+\sin(n)}{10^n} \leq \frac{2}{10^n}$$

By D.C.T. to $\sum_{n=0}^{\infty} \frac{2}{10^n} = \frac{2}{1-1/10}$ $\left(\begin{array}{c}\text{geometric} \\ \text{conv. since} \\ r = 1/10\end{array}\right)$ we find S converges.

§12.4#13) Conv / Div ?

$$S = \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n^{1.2}} \qquad \text{observe } \frac{\tan^{-1}(n)}{n^{1.2}} > 0 \text{ for } n > 1.$$

We can use the D.C.T, notice $\frac{\tan^{-1}(n)}{n^{1.2}} \leq \frac{2}{n^{1.2}}$

and $\sum_{n=1}^{\infty} \frac{2}{n^{1.2}} = 2\left(\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}\right)$ converges by $P = 1.2$ series test.

Thus S converges by D.C.T.

§12.4#16) Conv / Div ?

$$S = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$$, positive terms, can use D.C.T.

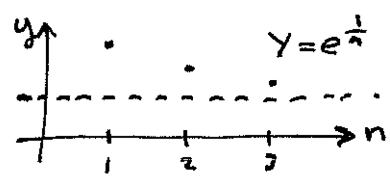
Observe that $\frac{1}{\sqrt{n^3+1}} < \frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}}$. Note $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ conv.

by the $p = 3/2$ series test. Hence S converges by D.C.T.

§12.4#28) Conv / Div ?

$$S = \sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$$, has positive terms, can use D.C.T.

Observe that $\frac{e^{1/n}}{n} \geq \frac{1}{n}$ since $e^{1/n} \geq 1$ for $n \geq 1$



But, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by $P = 1$ series test. Hence S diverges by the D.C.T.

§12.4 #30 Conv/Div?

$$S = \sum_{n=1}^{\infty} \frac{n!}{n^n} \quad , \text{ use ratio test.}$$

$$L = \lim_{n\to\infty} \left| \left(\frac{(n+1)!}{(n+1)^{n+1}}\right)\cdot\left(\frac{n^n}{n!}\right) \right|$$

$$= \lim_{n\to\infty} \left(\frac{\cancel{(n+1)}\,\cancel{n!}\,n^n}{\cancel{(n+1)}(n+1)^n\,\cancel{n!}} \right)$$

$$= \lim_{n\to\infty} \left[\left(\frac{n}{n+1} \right)^n \right]$$

Thus $\ln(L) = \lim_{n\to\infty} \left[\ln\left(\frac{n}{n+1}\right)^n \right]$

$$= \lim_{n\to\infty} \left[\frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}} \right]$$

$$\underset{(\frac{0}{0})}{\overset{\mathcal{L}}{=}} \lim_{n\to\infty} \left[\frac{\frac{n}{n+1}\cdot\frac{d}{dn}\left[\frac{n}{n+1}\right]}{\frac{-1}{n^2}} \right]$$

: ext. n to be continuous.

$$= \lim_{n\to\infty} \left[\left(\frac{n}{n+1}\right)(-n^2)\left(\frac{n+1-n}{(n+1)^2}\right) \right]$$

$$= \lim_{n\to\infty} \left[\frac{-n^3}{(n+1)^3} \right]$$

$$= -1 \qquad \therefore \quad L = e^{-1} < 1$$

Thus S converges by ratio test.

Remark: doubtless we could have used ratio test instead of the D.C.T. in a few problems in §12.4.