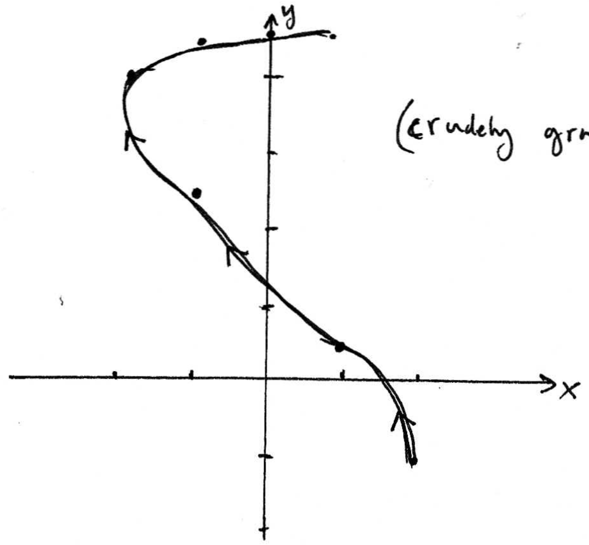
HOMEWORK 29: §11.1 # 2, 4, 6, 8, 12, 14, 18, 34, 35, 46

§11.1 #2 $x = 2\cos t$, $y = t - \cos t$, $0 \le t \le 2\pi$

t	$2\cos t$	$t - \cos t$	approx. pt.
0	2	-1	$(2,-1)$
$\pi/3$	1	$\frac{\pi}{3} - \frac{1}{2}$	$(1, 0.5)$
$\frac{\pi}{2}$	0	$\pi/2$	$(0, 1.5)$
$2\pi/3$	-1	$\frac{2\pi}{3} + \frac{1}{2}$	$(-1, 2.5)$
π	-2	$\pi + 1$	$(-2, 4)$
$4\pi/3$	-1	$\frac{4\pi}{3} + \frac{1}{2}$	$(-1, 4.5)$
$\frac{3\pi}{2}$	0	$\frac{3\pi}{2}$	$(0, 4.5)$
$5\pi/3$	1	$\frac{5\pi}{3} - \frac{1}{2}$	$(1, 4.5)$



(crudely graphing)

§11.1 #4 $x = e^{-t} + t$ and $y = e^t - t$, $-2 \le t \le 2$

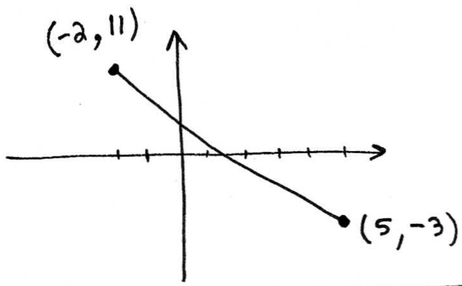
use technology to check answer.

§11.1 #6 Sketch curve & find Cartesian Eq^{n} for $x = 1 + t$, $y = 5 - 2t$ for $-3 \le t \le 4$

Solve for t since we can w/o much trouble here:

$$t = x - 1 = \frac{5-y}{2} \implies 5 - y = 2x - 2$$
$$\implies \boxed{y = -2x + 7 \text{ for } -2 \le x \le 5}$$

(note $-3 \le t \le 4$
$\implies -2 \le t + 1 \le 5$

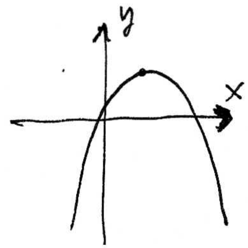


§11.1 #8 find Cartesian Eq^{n}'s for $x = 1 + 3t$, $y = 2 - t^2$

Notice $x - 1 = 3t$ and $t^2 = 2 - y$ thus

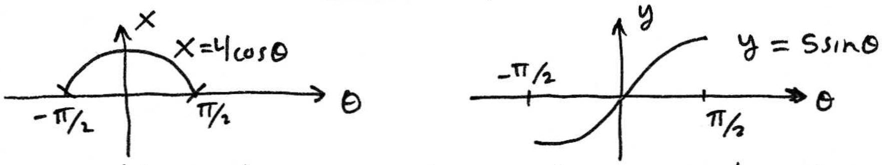
$$9t^2 = (x-1)^2 = 9(2-y) \quad \therefore \quad 9y = 18 - (x-1)^2$$
$$\implies \boxed{y = 2 - \frac{1}{9}(x-1)^2}$$

(parabola)

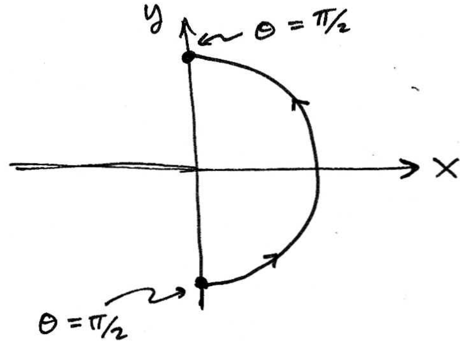


§11.1 #12 Find Cartesian Eq^{n}'s for $x = 4\cos\theta$, $y = 5\sin\theta$ for $-\pi/2 \leq \theta \leq \pi/2$ and sketch curve with arrows to indicate direction of increasing θ

Recall $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$, this is a portion of an ellipse.



We have (from the parametric eq^{n}'s and bound $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$) that $0 \leq x \leq 4$ whereas $-5 \leq y \leq 5$ hence,



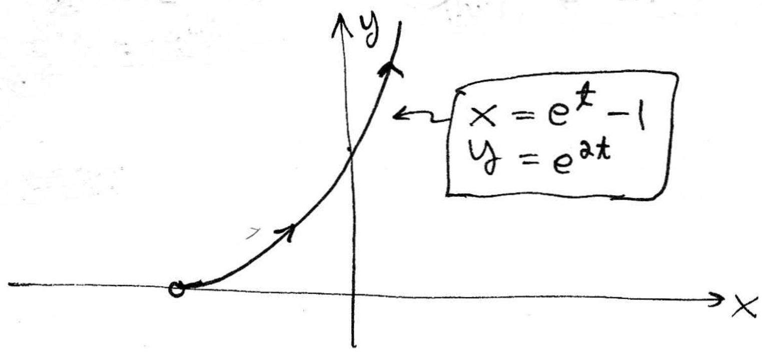
Remark: θ need not be the same as the polar standard angle. we could just as well write $x = 4\sin\theta$, $y = 5\cos\theta$ then $\theta \neq \theta_{polar}$. Just be warned you have to think about the parametric eq^{n}'s case by case basis.

§11.1 #14 Analyze $x = e^{t} - 1$ and $y = e^{2t}$ by eliminating the parameter to find Cartesian Eq^{n} for the curve. Note, since no bounds are placed on t it is assumed $-\infty < t < \infty$ is allowed

If $x = e^{t} - 1$ then $x + 1 = e^{t} \Rightarrow (x+1)^2 = (e^{t})^2 = e^{2t} = y$

thus $\boxed{y = (x+1)^2}$ parabola that bounces at $(-1, 0)$

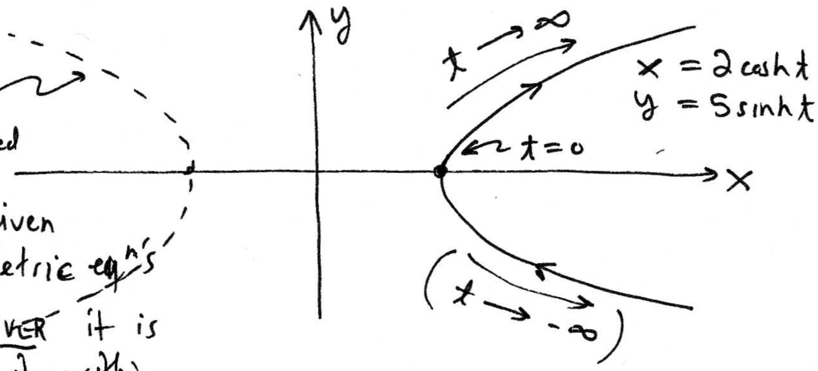
However, we only have part of the parabola since $x = e^{t} - 1 > -1$ and likewise $y = e^{2t} > 0$ thus we get the right-half of the parabola,



$\boxed{\S 11.1 \# 18}$ $x = 2\cosh t$, $y = 5\sinh t$; sketch

Note $\cosh^2 t - \sinh^2 t = 1$ thus $\underline{\dfrac{x^2}{4} - \dfrac{y^2}{25} = 1}$

We have a part of a hyperbola. Note that $\cosh(t) = \frac{1}{2}(e^t + e^{-t}) > 1$ for all $t \in \mathbb{R}$ thus we have selected the right half of this horizontally opening hyperbola;



not covered by the given parametric eq^{n}'s

HOWEVER it is implicit within sol^{n} set of $\dfrac{x^2}{4} - \dfrac{y^2}{25} = 1$

$\boxed{\S 11.1 \# 34 \quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \text{ find parametric eq}^{n}\text{'s for ellipse}}$

$x = a\cos(\theta)$, $y = b\sin\theta$ works since

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{(a\cos\theta)^2}{a^2} + \frac{(b\sin\theta)^2}{b^2} = \cos^2\theta + \sin^2\theta = 1.$$

(Many other choices possible)

$\underline{\S 11.1 \# 35}$ to get smiley face try graphing

I.) $x = 2 + 2\cos\theta$, $y = 2 + 2\sin\theta$

II.) $x = 1 + 0.1\cos\theta$, $y = 1 + 0.1\sin\theta$

III.) $x = 3 + 0.1\cos\theta$, $y = 3 + 0.1\sin\theta$

IV.) $x = 2 + \cos\theta$, $y = 2 + \sin\theta$

(let I, II, III graph for $0 \le \theta \le 2\pi$ and IV for $0 \le \theta \le \pi$.)

$\underline{\S 11.1 \# 46}$ much fun, we worked out in lecture this Fall 2009 semester.