HOMEWORK 3 : CALCULUS II : §8.2# 2, 7, 8, 16, 20, 25, 41, 43, 44 (STEWART 6th Ed.)

§8.2#2

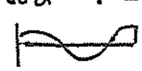
$$\int \sin^6 x \cos^3 x\, dx = \int \sin^6 x\,(1-\sin^2 x)\cos x\, dx$$
$$= \int u^2(1-u^2)(+du) \quad : \quad \begin{bmatrix} u = \sin x \\ du = \cos x\, dx \end{bmatrix}$$
$$= \int (u^2 - u^4)\, du$$
$$= \tfrac{1}{3}u^3 - \tfrac{1}{5}u^5 + C$$
$$= \boxed{\tfrac{1}{3}\sin^3 x - \tfrac{1}{5}\sin^5 x + C.}$$

§8.2#7

$$\int_0^{\pi/2} \cos^2\theta\, d\theta = \int_0^{\pi/2} \tfrac{1}{2}(1+\cos(2\theta))\, d\theta$$
$$= \tfrac{1}{2}\left(\theta + \tfrac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/2}$$
$$= \tfrac{1}{2}\left(\tfrac{\pi}{2} + \tfrac{1}{2}\sin(\pi)\right) - \tfrac{1}{2}\left(0 + \tfrac{1}{2}\sin(0)\right)$$
$$= \boxed{\pi/4.}$$

§8.2#8

$$\int_0^{\pi/2} \sin^2(2\theta)\, d\theta = \int_0^{\pi} \sin^2 u\, \frac{du}{2} \qquad \begin{bmatrix} u = 2\theta \\ du = 2d\theta \end{bmatrix} \Bigg| \begin{array}{l} u(\pi/2) = 2\frac{\pi}{2} = \pi. \\ u(0) = 0. \end{array}$$
$$= \tfrac{1}{4}\int_0^{\pi} (1-\cos(2u))\, du$$
$$= \boxed{\tfrac{\pi}{4}}$$

notice, this is periodic with period $T = \pi$ and cosine  has net signed-area zero over any period. This observation can save much labor if applied as needed.

§8.2#16 let $u = \sin\theta$

$$\int \cos^5(\sin\theta)\cos\theta\, d\theta = \int \cos^5 u\, du$$
$$= \int (1-\sin^2 u)^2 \cos u\, du$$
$$= \int (1-w^2)^2\, dw \quad : \quad [w = \sin u]$$
$$= \int [1 - 2w^2 + w^4]\, dw$$
$$= w - \tfrac{2}{3}w^3 + \tfrac{1}{5}w^5 + C = \sin(u) - \tfrac{2}{3}\sin^3 u + \tfrac{1}{5}\sin^5 u + C$$
$$= \boxed{\sin(\sin\theta) - \tfrac{2}{3}\sin^3(\sin\theta) + \tfrac{1}{5}\sin^5(\sin\theta) + C.}$$

§8.2 # 25) (note even # of sec, tan fncts ⇒ tan t = u subst.)

$$\int \sec^6 t\,dt = \int \sec^4 t \sec^2 t\,dt$$
$$= \int (1+\tan^2 t)^2 \sec^2 t\,dt$$
$$= \int (1+u^2)^2\,du \quad : \left[\begin{array}{l} u = \tan t \\ du = \sec^2 t\,dt \end{array}\right]$$
$$= \int (1+2u^2+u^4)\,du$$
$$= u + \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$$
$$= \boxed{\tan t + \frac{2}{3}\tan^3 t + \frac{1}{5}\tan^5 t + C.}$$

§8.2 # 41)

$$\int \csc(x)\,dx = \int \frac{-du}{u}$$
$$= -\ln|u| + C$$
$$= \boxed{-\ln|\csc(x)+\cot(x)| + C}$$

$$: \left[\begin{array}{l} u = \csc x + \cot x \\ du = [-\csc x \cot x - \csc^2 x]\,dx \\ \quad = -\csc x[\csc x + \cot x]\,dx \\ \quad = -u \csc x\,dx \\ \Rightarrow -\frac{du}{u} = \csc x\,dx \end{array}\right]$$

YES, this is the same as the text's answer $\int \csc x\,dx = \ln|\csc x - \cot x| + C$.
I challenge you to prove it.

§8.2 # 43) I'll use $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$ & $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ to derive the needed trig. identity.

$$\int \sin(8x)\cos(5x)\,dx = ?$$

Notice
$$\sin 8x \cos 5x = \frac{1}{4i}(e^{8ix} - e^{-8ix})(e^{5ix} + e^{-5ix})$$
$$= \frac{1}{4i}(e^{13ix} + e^{3ix} - e^{-3ix} - e^{-13ix})$$
$$= \frac{1}{2}\cdot\frac{1}{2i}(e^{13ix} - e^{-13ix}) + \frac{1}{2}\cdot\frac{1}{2i}(e^{3ix} - e^{-3ix})$$
$$= \frac{1}{2}\sin(13x) + \frac{1}{2}\sin(3x).$$

$$\int \sin(8x)\cos(5x)\,dx = \frac{1}{2}\int \sin(13x)\,dx + \frac{1}{2}\int \sin(3x)\,dx$$
$$= \boxed{\frac{-1}{26}\cos(13x) - \frac{1}{6}\cos(3x) + C}$$

$$: \left[\begin{array}{l} \text{implicit} \\ u = 13x \\ \text{and } u = 3x \\ \text{substitutions} \end{array}\right]$$

§8.2 # 44 Calculate $\int \cos \pi x \cos 4\pi x \, dx$. First observe

$$\cos 4\pi x \cos \pi x = \frac{1}{4}\left(e^{4\pi i x} + e^{-4\pi i x}\right)\left(e^{\pi i x} + e^{-\pi i x}\right)$$

$$= \frac{1}{4}\left(e^{5\pi i x} + e^{3\pi i x} + e^{-3\pi i x} + e^{-5\pi i x}\right)$$

$$= \frac{1}{2}\left[\frac{1}{2}\left(e^{5\pi i x} + e^{-5\pi i x}\right)\right] + \frac{1}{2}\left[\frac{1}{2}\left(e^{3\pi i x} + e^{-3\pi i x}\right)\right]$$

$$= \frac{1}{2}\cos(5\pi x) + \frac{1}{2}\cos(3\pi x).$$

Use this identity to see

$$\int \cos \pi x \cos 4\pi x \, dx = \frac{1}{2}\int \cos(5\pi x)\,dx + \frac{1}{2}\int \cos(3\pi x)\,dx$$

$$= \frac{1}{2}\int \cos(u)\,\frac{du}{5\pi} + \frac{1}{2}\int \cos(w)\,\frac{dw}{3\pi}$$

[I often omit these, you can as well.]

$$= \boxed{\frac{1}{10\pi}\sin(5\pi x) + \frac{1}{6\pi}\sin(3\pi x) + C}$$