§11.2 # 1) Find dy/dx given $x = t\sin t$, $y = t^2 + t$

$$\frac{dy}{dx}(t) = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \boxed{\frac{2t+1}{\sin t + t\cos t}}$$

§11.2 # 42) Let $x = e^t + e^{-t}$, $y = 5 - 2t$, $0 \le t \le 3$ find length of curve

Note $\frac{dx}{dt} = \frac{d}{dt}\left(2\cosh t\right) = 2\sinh t$ & $\frac{dy}{dt} = -2$.

Thus $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{4\sinh^2 t + 4} = \sqrt{4\cosh^2 t} = 2\cosh t$.

Thus $S = \int_0^3 2\cosh t \, dt = 2\sinh t \Big|_0^3 = \boxed{2\sinh(3) = e^3 - e^{-3}}$

§11.2 # 44) Let $x = 3\cos t - \cos 3t$, $y = 3\sin t - \sin 3t$, $0 \le t \le \pi$ find arc length

$dx/dt = -3\sin t + 3\sin(3t)$, $dy/dt = 3\cos t - 3\cos 3t$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 = 9\left(\sin^2 t - 2\sin t\sin 3t + \sin^2(3t)\right)$$

$$\Rightarrow \left(\frac{dy}{dt}\right)^2 = 9\left(\cos^2 t + 2\cos t\cos 3t + \cos^2(3t)\right)$$

Hence,

$$S = \int_0^\pi \sqrt{9\left[\sin^2 t - 2\sin t\sin 3t + \sin^2 3t + \cos^2 t + 2\cos t\cos 3t + \cos^2 3t\right]}\, dt$$

$$= \int_0^\pi 3\sqrt{2\left[\underbrace{\cos t\cos 3t - \sin t\sin 3t}_{\text{adding } \angle\text{'s formula}}\right]}\, dt$$

$$= \int_0^\pi 3\sqrt{2\left[\cos(4t) + 1\right]}\, dt$$

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$
$\Rightarrow 2\cos^2(2\theta) = 1 + \cos(4\theta)$

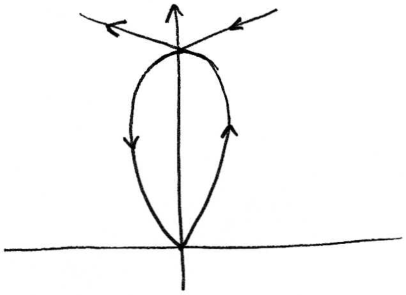
$$= \int_0^\pi 3\sqrt{2\left(2\cos^2(2t)\right)}\, dt$$

$$= 6\int_0^\pi |\cos(2t)|\, dt$$

$$= 24\int_0^{\pi/4} \cos(2t)\, dt = \frac{24}{2}\sin(2t)\Big|_0^{\pi/4} = \boxed{12}$$

§11.2 #48) Find length of loop $x = 3t - t^3$, $y = 3t^2$

After 10 minutes or so of tinkering one finds $-\sqrt{3} \le t \le \sqrt{3}$ and this loop looks roughly like



$$\frac{dx}{dt} = 3 - 3t^2 \longrightarrow \left(\frac{dx}{dt}\right)^2 = 9(1 - 2t^2 + t^4)$$

$$\frac{dy}{dt} = 6t \longrightarrow \left(\frac{dy}{dt}\right)^2 = 36t^2$$

$$S = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{9 - 18t^2 + 9t^4 + 36t^2}\, dt$$

$$= 2\int_0^{\sqrt{3}} \sqrt{9 + 18t^2 + 9t^4}\, dt$$

$$= 2\int_0^{\sqrt{3}} 3\sqrt{t^4 + 2t^2 + 1}\, dt$$

<u>note</u>: $t^4 + 2t^2 + 1 = (t^2+1)^2$.

$$= 2\int_0^{\sqrt{3}} 3\,(t^2 + 1)\, dt$$

$$= 6\left(\frac{t^3}{3} + t\right\Big|_0^{\sqrt{3}}$$

$$= 6\left(\frac{3\sqrt{3}}{3} + \sqrt{3}\right)$$

$$= \boxed{12\sqrt{3}.}$$

§11.2 #52) left to reader.