HOMEWORK 31 : § 9.1 # 9, 12, 38, 40

§9.1 #9 Find length of $y = x^5/6 + 1/10x^3$, $1 \le x \le 2$

$$S = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_1^2 \sqrt{1 + \left(\frac{5x^4}{6} - \frac{3}{10x^4}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{25}{36}x^8 - 2\left(\frac{15}{60}\right)\frac{x^4}{x^4} + \frac{9}{100x^8}}\, dx$$

$$= \int_1^2 \sqrt{\frac{1}{2} + \frac{25}{36}x^8 + \frac{9}{100x^8}}\, dx$$

$$= \int_1^2 \sqrt{\left(\frac{3}{10x^4} + \frac{5x^4}{6}\right)^2}\, dx$$

notice $2\left(\frac{3}{10}\right)\left(\frac{5}{6}\right) = 2\left(\frac{15}{60}\right) = \frac{1}{2}$

you can multiply this out to check that the miracle did indeed occur.

$$= \int_1^2 \left(\frac{3}{10x^4} + \frac{5}{6}x^4\right) dx$$

$$= \left(\frac{-1}{10x^3} + \frac{1}{6}x^5\right)\Big|_1^2$$

$$= \left(\frac{-1}{10(8)} + \frac{32}{6}\right) - \left(\frac{-1}{10} + \frac{1}{6}\right)$$

$$= \frac{1}{10}\left[1 - \frac{1}{8}\right] + \frac{1}{6}\left[32 - 1\right]$$

$$= \frac{7}{80} + \frac{31}{6}$$

$$= \frac{21 + 1231}{240}$$

$$= \frac{1252}{240}$$

$$= \frac{626}{120}$$

$$= \boxed{\frac{313}{60}}$$

§9.1 #12 $y = \ln(\cos(x))$ $0 \le x \le \pi/3$

Note $dy/dx = \frac{-\sin x}{\cos x} = -\tan x$ $\therefore$ $ds = \sqrt{1 + (y')^2}\, dx = \sec(x)\, dx$

$$\therefore\ s = \int_0^{\pi/3} \sec(x)\, dx = \ln|\sec x + \tan x|\Big|_0^{\pi/3}$$

$$= \ln(\sec(\pi/3) + \tan(\pi/3)) - \ln(\sec(0) + \tan(0))$$

$$= \boxed{\ln(2 + \sqrt{3})}$$

$\tan \pi/3 = \frac{\sin \pi/3}{\cos \pi/3} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$