HOMEWORK 8 : §8.8 # 2, 6, 9, 16, 17, 18, 27, 31, 35, 38, 78 (STEWART 6th Ed.)

no sol'ns (did in lecture)

§8.8 #2)

a.) $\int_1^2 \frac{dx}{2x-1}$ is proper integral since $\frac{1}{2x-1}$ is continuous on $[1,2]$.

b.) $\int_0^1 \frac{dx}{2x-1}$ is improper since $\frac{1}{2x-1}$ has V.A. at $x = 1/2$

c.) $\int_{-\infty}^{\infty} \frac{\sin(x)}{1+x^2}\,dx$ is improper due to its bounds of $\pm\infty$.

d.) $\int_1^2 \ln(x-1)\,dx$ is improper since $\ln(x-1)$ is not bounded at $x=1$, it has V.A. at that point

§8.8 #6)

$$\int_{-\infty}^{0} \frac{dx}{2x-5} = \lim_{t\to-\infty} \int_t^0 \frac{dx}{2x-5}$$

$$= \lim_{t\to-\infty} \left[\left(\tfrac{1}{2}\ln|2x-5|\right)\Big|_t^0 \right]$$

$$= \lim_{t\to-\infty} \left[\tfrac{1}{2}\ln|-5| - \tfrac{1}{2}\ln\underbrace{|2t-5|}_{\infty} \right]$$

$$= \boxed{-\infty}$$

§8.8 #9)

$$\int_4^{\infty} e^{-y/2}\,dy = \lim_{t\to\infty} \int_4^t e^{-\frac{y}{2}}\,dy$$

$$= \lim_{t\to\infty} \left[\frac{e^{-\frac{t}{2}}}{-1/2} - \frac{e^{-4/2}}{-1/2} \right]$$

$$= \lim_{t\to\infty} \left[-\tfrac{1}{2}\underbrace{e^{-t/2}}_{0} + 2e^{-2} \right] = \boxed{\frac{2}{e^2}}.$$

§8.8 #16)

$$\int_{-\infty}^{\infty} \cos(\pi t)\,dt = \lim_{t\to-\infty}\int_{t}^{0}\cos \pi t\,dt + \lim_{\lambda\to\infty}\int_{0}^{\lambda}\cos \pi t\,dt$$

$$= \underbrace{\lim_{t\to-\infty}\left(\frac{-1}{\pi}\sin \pi t\right)}_{\text{d.n.e.}} + \underbrace{\lim_{\lambda\to\infty}\left(\frac{1}{\pi}\sin \pi \lambda\right)}_{\text{d.n.e.}}$$

integral diverges.

§8.8 #17)

$$\int_{1}^{\infty}\frac{x+1}{x^2+2x}\,dx = \lim_{t\to\infty}\frac{1}{2}\int_{1}^{t}\left(\frac{1}{x}+\frac{1}{x+2}\right)dx$$

$$\frac{x+1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$x+1 = A(x+2) + Bx$$

$x=0$) $1 = 2A \quad \therefore A = 1/2$

$x=-1$) $0 = A - B \quad \therefore B = 1/2$

$$= \lim_{t\to\infty}\left(\frac{1}{2}\ln|t| + \frac{1}{2}\ln|t+2| - \ln|1| - \ln|3|\right)$$

$$= \lim_{t\to\infty}\left(\frac{1}{2}\ln(t(t+2)) - \ln(3)\right)$$

$$= \boxed{\infty.}$$

§8.8 #18)

$$\int_{0}^{\infty}\frac{dz}{z^2+3z+2} = \lim_{t\to\infty}\int_{0}^{t}\left(\frac{1}{z+1}-\frac{1}{z+2}\right)dz$$

$$\frac{1}{z^2+3z+2} = \frac{A}{z+1} + \frac{B}{z+2}$$

$$1 = A(z+2) + B(z+1)$$

$z=-2$) $1 = -B$

$z=-1$) $1 = A$

$$= \lim_{t\to\infty}\left[\ln(t+1) - \ln(t+2) - \ln(1) + \ln(2)\right]$$

$$= \lim_{t\to\infty}\left[\ln\left(\frac{t+1}{t+2}\right) + \ln(2)\right]$$

$$= \lim_{t\to\infty}\left[\ln\left(\frac{1+1/t}{1+2/t}\right) + \ln(2)\right]$$

$$= \ln\left(\frac{1+0}{1+0}\right) + \ln(2)$$

$$= \boxed{\ln(2).}$$

§8.8 #27

$$\int_0^1 \frac{3}{x^5}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{3\,dx}{x^5}$$

$$= \lim_{t\to 0^+} \left.\frac{-3}{4x^4}\right|_t^1$$

$$= \lim_{t\to 0^+} \left[\frac{-3}{4} + \underbrace{\frac{3}{4t^4}}_{\to\infty}\right]$$

$$= \boxed{\infty.}$$

§8.8 #31

$$\int_{-2}^{3} \frac{dx}{x^4} = \lim_{t\to 0^-} \int_{-2}^{t} \frac{dx}{x^4} + \lim_{s\to 0^+} \int_s^3 \frac{dx}{x^4}$$

$$= \lim_{t\to 0^-} \left(\left.\frac{-1}{3x^3}\right|_{-2}^{t} \right) + \lim_{s\to 0^+} \left(\left.\frac{-1}{3x^3}\right|_s^3 \right)$$

$$= \lim_{t\to 0^-} \Big(\underbrace{\frac{-1}{3t^3}}_{+\infty} + \frac{1}{24} \Big) + \lim_{s\to 0^+} \Big(\frac{-1}{81} + \underbrace{\frac{1}{3s^3}}_{\infty} \Big)$$

$$= \boxed{\infty.}$$

§8.8 #35

this part continuous, no need to lump it into the improper part.

$$\int_0^3 \frac{dx}{x^2-6x+5} = \frac{1}{4}\int_0^3 \left(\frac{1}{x-5} - \frac{1}{x-1} \right) dx$$

$$= \frac{1}{4}\int_0^3 \frac{dx}{x-5} - \frac{1}{4}\int_0^3 \frac{dx}{x-1}$$

$$= \frac{1}{4} \ln|x-5|\Big|_0^3 - \frac{1}{4}\left[\lim_{t\to 1^-} \int_0^t \frac{dx}{x-1} + \lim_{\lambda\to 1^+} \int_\lambda^3 \frac{dx}{x-1} \right]$$

$$= \frac{1}{4}\big(\ln(2) - \ln(5)\big) - \frac{1}{4}\Big[\lim_{t\to 1^-} \big(\underbrace{\ln|t-1|}_{-\infty} \big) - \lim_{\lambda\to 1^+} \big(\underbrace{\ln|\lambda-1|}_{-(-\infty)} + \ln|\ldots$$

$$\frac{1}{x^2-6x+5} = \frac{A}{x-1} + \frac{B}{x-5}$$

$$1 = A(x-5) + B(x-1)$$

$x=1$: $1 = -4A$

$x=5$: $1 = 4B$

$\boxed{\text{DIVERGES}}$