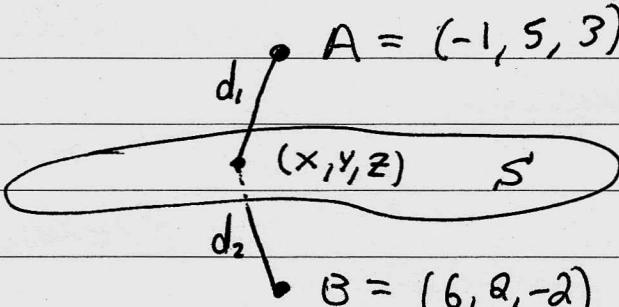


(1)

MATH 231, 8/21/08 LECTURE SUMMARY

- I started with a homework question. §13.1 #39



What is S?

Pick a point

(x, y, z) on S
then we have

$$d_1 = d_2$$

$$\sqrt{(x+1)^2 + (y-5)^2 + (z-3)^2} = \sqrt{(x-6)^2 + (y-2)^2 + (z+2)^2}$$

$$\Rightarrow x^2 + 2x + 1 + y^2 - 10y + 25 + z^2 - 6z + 9 = \rightarrow$$

$$\rightarrow = x^2 - 12x + 36 + y^2 - 4y + 4 + z^2 + 4z + 4$$

The quadratic terms all cancel leaving only first order terms,

$$14x - 6y - 10z = 9$$

We will learn Tuesday this is a plane with normal $\langle 14, -6, -10 \rangle = N$. (I would not have expected you to know that if I had collected it)

(2)

- Next, we discussed the dot product.

We went over $\boxed{E3}$ and $\boxed{E4}$ (with $\alpha = 1$).

- A question was raised as to what the purpose of unit vectors was. Why bother with $\hat{\vec{A}}$ for \vec{A} (Recall $\hat{\vec{A}} = \frac{1}{|\vec{A}|} \vec{A}$, it is the vector of length one that points along \vec{A})
We discussed pg. $\boxed{244}$ as the answer in essence.

(3)

- At some point I mentioned

$$\textcircled{246} \Rightarrow \vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\textcircled{250} \Rightarrow \vec{A} \times \vec{B} = AB \sin \theta \hat{n}, \hat{n} \text{ given by RH-rule.}$$

both of these relations require some moderately difficult calculation to establish their validity.

- CORRECTION! I believe I said that

$$\vec{A} \times \vec{B} = (AB \sin \theta) \hat{A} \times \hat{B} \text{ at some time.}$$

This is incorrect. I thought $|\hat{A} \times \hat{B}| = 1$

but this is not generally true. For

$$\text{example, } \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \rangle = \hat{A} \text{ and}$$

$$\langle 1, 0, 0 \rangle = \hat{B} \text{ yields}$$

$$\hat{A} \times \hat{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 \end{vmatrix} = \hat{k} \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 \end{vmatrix} = -\frac{1}{\sqrt{2}} \hat{k}$$

$$\text{so } |\hat{A} \times \hat{B}| = \frac{1}{\sqrt{2}} \neq 1.$$

This error unknowingly poisoned my derivation of V for E9. Notice Lagranges Identity stated on 250 says

$$|\vec{A} \times \vec{B}|^2 = \begin{vmatrix} \vec{A} \cdot \vec{A} & \vec{A} \cdot \vec{B} \\ \vec{A} \cdot \vec{B} & \vec{B} \cdot \vec{B} \end{vmatrix}$$

thus only in the case $\hat{A} \cdot \hat{B} = 0$ does $|\vec{A} \times \vec{B}| = 1$.

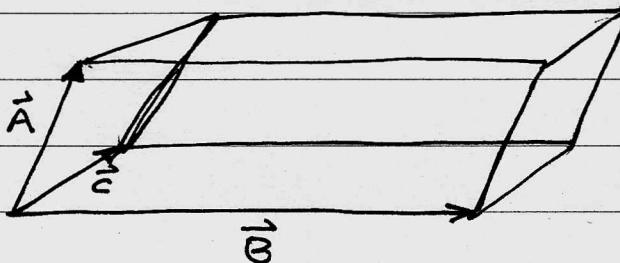
Unfortunately, I had such a case in mind

when I wrote the bogus $\vec{A} \times \vec{B} = AB \sin \theta (\hat{A} \times \hat{B})$.

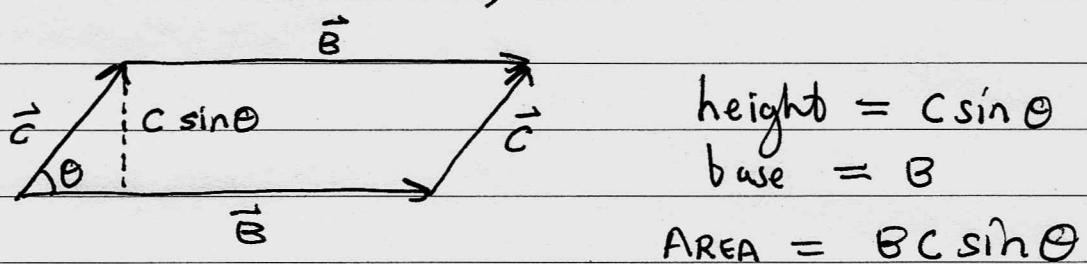
(I was thinking $\hat{i}, \hat{j}, \hat{k}$.)

(4)

- We covered E6 and E9. I got stuck on E9 because of the error I mentioned on ③. Let me clean up the explanation of why $V = |\vec{A} \cdot (\vec{B} \times \vec{C})|$



Look at the base of this to find the typical cross-sectional area,



To find volume we need only multiply AREA by the altitude of the object since the AREA is constant for each slice // to the base of the shape. Notice $\vec{B} \times \vec{C} = BC \sin \theta \hat{n}$ where \hat{n} points straight up off the base. (its normal to the \vec{B}, \vec{C} containing plane.)

$\text{comp}_{\hat{n}}(\vec{A}) = \vec{A} \cdot \hat{n}$
gives length of the part of \vec{A} which is normal to the cross-section.

$$V = (\vec{A} \cdot \hat{n}) BC \sin \theta = |\vec{A} \cdot (BC \sin \theta \hat{n})| = |\vec{A} \cdot (\vec{B} \times \vec{C})|.$$

(5)

Remark: I drew $\vec{A}, \vec{B}, \vec{C}$ so that $\vec{A} \cdot (\vec{B} \times \vec{C})$ is positive. That need not be the case. If $\vec{A}, \vec{B}, \vec{C}$ are otherwise arranged it might happen $\vec{A} \cdot (\vec{B} \times \vec{C}) = -V$ so taking the absolute value fixes this.

- Last 10 minutes of lectures I tried to get across a little about the Einstein Summation Notation.

I'll try again Tuesday. But much much slower, and with motivation.

Sorry folks, didn't mean to scare you.

(See (H9), (H10), (H11) if you want a)
preview of Tuesdays

- Homework from §13.5 moved back a day due to timing issues.

- Tuesday Schedule:

Einstein Σ notation

 Lines / Planes

 Functions, Graphing

 with Mathematica

 Homework Quiz ?

 (maybe at start of class)

 don't be late.