These problems are worth 1pt a piece at least. Feel free to use Mathematica or some other CAS to illustrate as needed. If I have derived a normal vector field in the notes or lecture then you may use it, but please mention the source of the fact.

Problem 205 Show that for a simply connected region R with consistently oriented boundary ∂R if f, g are differentiable on some open set containing R then

$$\iint_R \big(f\nabla^2 g + \nabla f \bullet \nabla g\big) dA = \int_{\partial R} f\nabla g \bullet \widehat{n}\, ds.$$

Problem 206 Show that for a simply connected region R with consistently oriented boundary ∂R if f, g are differentiable on some open set containing R then

$$\iint_R \big(f\nabla^2 g - g\nabla^2 f\big) dA = \int_{\partial R} \big[f\nabla g \bullet \widehat{n} - g\nabla f \bullet \widehat{n}\big]\, ds.$$

Problem 207 Suppose $\nabla^2 f = 0$ on a simply connected region R. If $f|_{\partial R} = 0$ then what can you say about f throughout R?
(here $|_{\partial R}$ denotes restriction of f to the subset ∂R. In particular this means you are given that $f(x, y) = 0$ for all $(x, y) \in \partial R$.)

Problem 208 Suppose $b : \mathbb{R}^2 \to \mathbb{R}$ is a particular function and $\nabla^2 f = b$ on a simply connected region R. If g is another such solution ($\nabla^2 g = b$) on R then show that $f = g$ on R.
The equation $\nabla^2 f = b$ is called Poisson's Equation. When $b = 0$ then it's called Laplace's Equation. You are showing the solution to Poisson's Equation is unique on a simply connected region. **Hint:** *use the last problem's result on $f - g$... hmmm... I guess this is a retroactive hint for Problem 207 if you think about it.*

Problem 209 Find the surface area of $z = xy$ for $x^2 + y^2 \leq 1$.

Problem 210 Find the surface area of the plane $y + 2z = 2$ bounded by the cylinder $x^2 + y^2 = 1$.

Problem 211 Find the surface area of the cone frustrum $z = \frac{1}{3}\sqrt{x^2 + y^2}$ with $1 \leq z \leq 4/3$

Problem 212 Find the surface area of torus with radii $A, R > 0$ and $R \geq A$ parametrized by

$$\vec{X}(\alpha, \beta) = \Big\langle \big[R + A\cos(\alpha)\big]\cos(\beta), \big[R + A\cos(\alpha)\big]\sin(\beta), A\sin(\alpha) \Big\rangle$$

for $0 \leq \alpha \leq 2\pi$ and $0 \leq \beta \leq 2\pi$.

Problem 213 Find an explicit double integral which gives the surface area of the graph $x = g(y, z)$ for $(y, z) \in D$.

Problem 214 Consider a napkin ring which is formed by taking a sphere of radius R and drilling out a circular cylinder of radius B through the center of the sphere. Find the surface area of the napkin ring (include the inner as well as outer surfaces).

Problem 215 Integrate $H(x, y, z) = xyz$ over the surface of the solid $[0, a] \times [0, b] \times [0, c]$ where $a, b, c > 0$.

Problem 216 Integrate $G(x, y, z) = x^2$ over the surface of the unit-sphere.

Problem 217 Integrate $H(x, y, z) = z - x$ on the graph $z = x + y^2$ over the triangular region with vertices $(0, 0, 0), (1, 0, 0)$ and $(0, 1, 0)$.

Problem 218 Consider a thin-shell of constant density δ. Let the shell be cut from the cone $x^2 + y^2 - z^2 = 0$ by the planes $z = 1$ and $z = 2$. Find **(a.)** the center of mass and **(b.)** the moment of intertia with respect to the z-axis.

Problem 219 Find the flux of $\vec{F}(x,y,z) = \langle z^2, x, -3z\rangle$ through the parabolic cylinder $z = 4 - y^2$ bounded by the planes $x = 0$, $x = 1$ and $z = 0$. Assume the orientation of the surface is outward, away from the x-axis.

Problem 220 Find the flux of $\vec{F}(x,y,z) = z\,\widehat{\mathbf{z}}$ through the portion the sphere of radius R in the first octant. Give the sphere an orientation which points away from the origin. In other words, assume the sphere is outwardly oriented.

Problem 221 Find the flux of $\vec{F}(x, y, z) = \langle -x, -y, z^2 \rangle$ through the conical frustrum $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$ with outward orientation.

Problem 222 Let S be the outward oriented paraboloid $z = 6 - x^2 - y^2$ for $x, z \geq 0$. Calculate the flux of $\vec{F} = (x^2 + y^2)\,\widehat{\mathbf{z}}$

Problem 223 Find the flux of $\vec{F}(x,y,z) = \langle 2xy, 2yz, 2xz \rangle$ upward through the subset of the plane $x+y+z = 2c$ where $(x,y) \in [0,c] \times [0,c]$.

Problem 224 Suppose $\vec{C}$ is a constant vector. Let $\vec{F}(x,y,z) = \vec{C}$ find the flux of $\vec{F}$ through a surface S on plane with nonzero vectors $\vec{A}, \vec{B}$. In particular, the surface S is parametrized by $\vec{r}(u,v) = \vec{r}_o + u\vec{A} + v\vec{B}$ for $(u,v) \in \Omega$.

Problem 225 Let $\vec{F}(x,y,z) = \langle a,b,c\rangle$ for some constants a,b,c. Calculate the flux of $\vec{F}$ through the upper-half of the outward oriented sphere $\rho = R$.

Problem 226 Once more consider the constant vector field $\vec{F}(x,y,z) = \langle a,b,c\rangle$. Calculate the flux of $\vec{F}$ through the downward oriented disk $z = 0$ for $\phi = \pi/2$.

Problem 227 Let $\vec{F} = \langle x^2, y^2, z^2 \rangle$. Calculate the flux of $\vec{F}$ through $z = 4 - x^2 - y^2$ for $z \geq 0$.

Problem 228 Let $\vec{F} = \langle x^2, y^2, z^2 \rangle$. Calculate the flux of $\vec{F}$ through the downward oriented disk $x^2 + y^2 \leq 4$ with $\phi = \pi/2$.

Problem 229 Let $\phi = \pi/4$ define a closed surface S with $0 \leq \rho \leq 2$. Find the flux of

$$\vec{F}(\rho, \phi, \theta) = \phi^2 \widehat{\rho} + \rho \widehat{\phi} + \widehat{\theta}$$

through the outward oriented S.

Problem 230 Consider the closed cylinder $x^2 + y^2 = R^2$ for $0 \leq z \leq L$. Find the flux of

$$\vec{F}(r, \theta, z) = \theta\,\widehat{\mathbf{z}} + z\widehat{\theta} + r^2\widehat{\mathbf{r}}$$

out of the cylinder.

Problem 231 Let S be the pseudo-tetrahedra with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$. Three of the faces of S_1 are subsets of the coordinate planes call these S_{xy}, S_{zx}, S_{yz} with the obvious meanings and call S_T the top face. Let $\vec{F} = \langle y, -x, y \rangle$ and define $\vec{G} = \nabla \times \vec{F}$.

(a.) Calculate the circulation of $\vec{F}$ around each face .

(b.) Calculate the flux of $\vec{G}$ through each face.

(c.) do you see any pattern?

Problem 232 Let $\vec{F}(x,y,z) = \langle x^2, 2x, z^2 \rangle$. Calculate $\int_E \vec{F} \bullet d\vec{r}$ where E is the CCW oriented ellipse $4x^2 + y^2 = 4$ with $z = 0$. (use Stoke's Theorem)

Problem 233 Let $\vec{F}(x,y,z) = \langle y^2 + z^2, x^2 + z^2, x^2 + y^2 \rangle$. Find the work done by $\vec{F}$ around the CCW (as viewed from above) triangle formed from the intersection of the plane $x + y + z = 1$ and the coordinate planes. (use Stoke's Theorem)

Problem 234 Let $\vec{F}(x, y, z) = \langle y^2 + z^2, x^2 + y^2, x^2 + y^2 \rangle$. Find the work done by $\vec{F}$ around the CCW-oriented square bounded by $x = \pm 1$ and $y = \pm 1$ in the $z = 0$ plane (use Stoke's Theorem).

Problem 235 Consider the elliptical shell $4x^2 + 9y^2 + 36z^2 = 36$ with $z \geq 0$ and let

$$\vec{F}(x, y, z) = \left\langle y, x^2, (x^2 + y^4)^{\frac{3}{2}} \sin(exp(\sqrt{xyz})) \right\rangle.$$

Find the flux of $\nabla \times \vec{F}$ through the outwards oriented shell.

Problem 236 Let $\vec{F} = \langle 2x, 2y, 2z \rangle$ and suppose S is a simply connected surface with boundary ∂S a simple closed curve. Show by Stoke's theorem that $\int_{\partial S} \vec{F} \bullet d\vec{r} = 0$

Problem 237 Suppose S is the union of the cylinder $x^2+y^2 = 1$ for $0 \leq z \leq 1$ and the disk $x^2+y^2 \leq 1$ at $z = 1$. Suppose $\vec{F}$ is a vector field such that

$$\nabla \times \vec{F} = \left\langle \sinh(z)(x^2+y^2), ze^{xy+\cos(x+y)}, (xz+y)\tan^{-1}(z) \right\rangle.$$

Calculate the flux of $\nabla \times \vec{F}$ though S.

Problem 238 Let E be the cube $[-1,1]^3$. Calculate the flux through ∂E of the vector field

$$\vec{F}(x,y,z) = \left\langle y-x, z-y, y-x \right\rangle$$

(please use the divergence theorem!)

Problem 239 Let E be the set of (x,y,z) such that $x^2+y^2 \leq 4$ and $0 \leq z \leq x^2+y^2$. Find the flux through ∂E of the vector field $\vec{F}$ given below:

$$\vec{F}(x,y,z) = \langle y, xy, -z \rangle$$

(please use the divergence theorem)

Problem 240 Suppose E is the spherical shell $R_1 \leq \rho \leq R_2$ and suppose $\vec{F}(x, y, z) = \nabla \times \vec{A}$ for some everywhere smooth vector field $\vec{A}$. Show that the flux through $\rho = R_1$ is the same as the flux through $\rho = R_2$ by applying the divergence theorem to the spherical shell.