It is doubtful that these will be returned before the first test. However, this material is covered by Test 1. Therefore, if you wish to study the verity of your answers you ought to keep some copy of your solution to compare against the solution I will hopefully post Tuesday. Thanks!

Problem 26 Show that a triangle inscribed in a semicircle must be a right triangle. To say the triangle is inscribed in a semicircle means one leg of the triangle lies on the diameter of the circle.

Problem 27 Find the point where the line $\vec{r}(t) = \langle 1+t, 2-3t, 3+4t\rangle$ intersects the plane $x+y+z=8$.

Problem 28 What is the distance between the line $\vec{r}_1(t) = \langle 1+t, 2-3t, 3+4t\rangle$ and $\vec{r}_2(t) = \langle 1+2t, 2+t, 3t\rangle$? Note: when discussing the distance between extended objects it is customary to define the distance between objects as the closest possible distance.

Problem 29 Simplify: $[\vec{A}+\vec{B}] \bullet (\vec{A}\times\vec{B})$.

Problem 30 Simplify: $[\vec{A}-\vec{B}] \bullet [\vec{A}+\vec{B}]$. If you are given that $\vec{A}-\vec{B}, \vec{A}+\vec{B}$ are orthogonal then what condition must this pair of vectors satisfy?

Problem 31 Use a computer graphing system to plot each of the curves below and include a printout in your homework. Comment on the identity of each curve (as in: "this is a circle, ellipse, hyperbola, spiral etc...")

(a.) let $x = \cos(t)$, $y = \sin(t)$ and $z = \cos(t)$ for $t \in [0, 2\pi]$

(b.) let $x = \cosh(t), y = 4\sinh(t)$ for $t \in \mathbb{R}$

(c.) let $x = e^{-t}\cos(50t)$ and $y = e^{-t}\sin(50t)$ and $z = e^{-t}\cos(50t)$ for $t \in [0, 1]$

(d.) let $x = e^t$, $y = 2e^t$ and $z = 3e^t$ for $t \in [0, \ln(3)]$

Problem 32 Identify each surface below. Either use results from my notes and simply state the name of the surface, or use a computer to plot the surface and print your result to justify your claim. (at least one of the surfaces below avoids explicit tabulation by my notes)

(a.) $z = x^2 + y^2$

(b.) $x^2 + y^2 - 3z^2 = 1$

(c.) $(x-1)^2 + (y+2)^2 + (z-3)^2 = 1$

(d.) $x^2 + 2y^2 = 1$

(e.) $x^2 + y^2 + z^2 + 2xy + 2xz = 1$

Problem 33 Provide parametrizations of the surfaces (a,b,c,d) given in the preceding problem. (or, provide a patch for part of the surface and explain where your patch covers, you must explicitly state the domain of your parameters for full credit, thanks!)

Problem 34 Parametrize the subset of the plane $x + 3y - z = 10$ for which $1 \le x \le 3$ and $2 \le y \le 4$.

Problem 35 Parametrize the subset of the plane $x + 3y - z = 10$ for which $1 \le y^2 + z^2 \le 4$.

Problem 36 Convert the following inequalities/equations to either spherical or cylindrical coordinates as appropriate and comment on the result.

(a.) $1 \leq x^2 + y^2 + z^2 \leq 3$

(b.) $0 \leq x^2 + y^2 \leq 4$

Problem 37 Consider the point $P = (\sqrt{3}, 1, 2)$. Find the

(a.) cylindrical coordinates of P

(b.) spherical coordinates of P

Problem 38 Write the vector $\vec{v} = \langle 1, 2, 3 \rangle$ in terms of the spherical frame at an arbitrary point with spherical coordinates ρ, ϕ, θ. In other words, find functions a, b, c of spherical coordinates such that $\vec{v} = a\widehat{\rho} + b\widehat{\phi} + c\widehat{\theta}$.
Hint: dot-products with respect to $\widehat{\rho}, \widehat{\phi}, \widehat{\theta}$ nicely isolate a, b and c if you make use of the fact that $\widehat{\rho}, \widehat{\phi}, \widehat{\theta}$ forms an orthonormal frame.

Problem 39 Find and parametrize the curve of intersection of $x^2 + y^2 = 4$ and $z = x^2 - y^2$.

Problem 40 Find and parametrize the curve of intersection of $x + y + z = 10$ and $z = x^2 + y^2$.

Problem 41 Convert the equation $4 = \rho \sin \phi$ to

(a.) cylindrical coordinates

(b.) cartesian coordinates

Problem 42 Find the intersection of $\phi = \pi/3$ and $z = 4$ and provide a parametrization which covers this curve of intersection.

Problem 43 Find the intersection of $\theta = \pi/4$ and $\rho = 4$ and provide a parametrization which covers this curve of intersection.

Problem 44 Find the polar coordinate equations for

(a.) the ellipse $x^2/a^2 + y^2/b^2 = 1$,

(b.) the line $y = 1 - 2x$

Problem 45 I recommend the use of vector arguments ($||\vec{c}||^2 = \vec{c} \bullet \vec{c}$ etc..) to analyze the following:

(a.) let $\vec{a} \neq 0$, characterize all vectors $\vec{r} = \langle x, y \rangle$ such that $||\vec{r} - \vec{a}|| = ||\vec{r} + \vec{a}||$

(b.) let $\vec{a} \neq 0$, characterize all vectors $\vec{r} = \langle x, y, z \rangle$ such that $||\vec{r} - \vec{a}|| = ||\vec{r} + \vec{a}||$

Problem 46 Calculate the following:

(a.) $\frac{d}{dt}\langle t^2, e^t, \ln(t) \rangle$

(b.) $\frac{d}{dt}\langle \cosh(t^2), \sinh(\ln(t)) \rangle$

(c.) $\int \langle 1, t, \sin(t) \rangle dt$

Problem 47 Let $\vec{g}, \vec{r}_o, \vec{v}_o$ be constant vectors. Let $\vec{r}(t) = \vec{r}_o + t\vec{v}_o + \frac{1}{2}t^2\vec{g}$ and calculate:

(a.) $\frac{d}{dt}\left[\, \vec{r} \,\right]$

(b.) $\frac{d^2}{dt^2}\left[\, \vec{r} \,\right]$

(c.) $\frac{d^3}{dt^3}\left[\, \vec{r} \,\right]$

Problem 48 Suppose $\vec{r}(t) = \langle \cos(\pi t), \sin(\pi t), t/8 \rangle$ for $t \geq 0$.

(a.) How many revolutions does this helix make at it travels from $z = 0$ to $z = 1$.

(b.) Calculate $d\vec{r}/dt$

(c.) Find the arclength of this curve from $z = 0$ to $z = 1$.

Problem 49 Find the total arclength of $\vec{r}(t) = 2\cos(t)\,\widehat{\mathbf{x}} + t\,\widehat{\mathbf{y}} + 2\sin(t)\,\widehat{\mathbf{z}}$ for $0 \leq t \leq 4\pi$. Also, find the arclength function and reparametrize this helix in terms of the arclength.

Problem 50 Find the arclength functions for $t \geq 0$ for:

(a.) $\vec{r}(t) = \langle e^{-t}, 1 - e^{-t} \rangle$

(b.) $\vec{r}(t) = (2 - 3t)\,\widehat{\mathbf{x}} + (1 + t)\,\widehat{\mathbf{y}} - 4t\,\widehat{\mathbf{z}}$