It is doubtful that these will be returned before the first test. However, this material is covered by Test 1. Therefore, if you wish to study the verity of your answers you ought to keep some copy of your solution to compare against the solution I will hopefully post Tuesday. Thanks!

I have marked the problems I anticipate as challenging with a *. You are free to use Mathematica (except for Problems 67 and 68) to perform tedious derivatives on the homework set. But, you must give credit when you do such. These problems are worth 1pt a piece at least.

Problem 51 Let R be a fixed positive constant. Suppose $\vec{r}(t) = \langle R\cos(t), 42, R\sin(t)\rangle$ for $0 \leq t \leq 2\pi$. Calculate, and simplify, the tangent, normal and binormal vector fields for the given path.

Problem 52 Calculate the curvature of the curve given in Problem 51.

Problem 53 Calculate the torsion of the curve given in Problem 51.

Problem 54 Suppose . $x = e^{-t}\cos(t)$ and $y = e^{-t}\sin(t)$ and $z = e^{-t}$ for $0 \leq t \leq 4\pi$. Calculate and simplify the tangent, normal and binormal vector fields for the curve parametrized by the given scalar parametric equations.

Problem 55 Calculate the curvature of the curve given in Problem 54.

Problem 56 Calculate the torsion of the curve given in Problem 54.

Problem 57 Find the point on the curve $y = 1/x$ for which the curvature is maximized. You may focus your efforts on the part of the curve with $x > 0$. (you can use the formula from Stewart for curvature if you wish... I'll probably attack it from my notes, it is likely the standard $y = f(x)$ curvature formula makes this easier)

Problem 58 * Suppose it is given that the three dimensional vectors $\vec{A}, \vec{B}$ are orthogonal vectors with $A = B = 1$. Show that if $\vec{A} \times \vec{B} = \vec{C}$ then $\vec{B} \times \vec{C} = \vec{A}$ and $\vec{C} \times \vec{A} = \vec{B}$.

Problem 59 * Show that $c_{12} = -c_{21}$ as defined in the discussion on pages 112-113 of my notes.

Problem 60 Suppose that you are given a path with torsion which is identically zero at all points on the path. Show that this path parametrizes a curve in a plane.

Problem 61 * A force is said to be central if it is directed along the line connecting a central point and has a magnitude which depends on on the distance from the center. For convenience put this force center at the origin thus we have $\vec{F}(x, y, z) = F(\rho)\widehat{\rho}$. If a mass m is subject to this central force alone then show that $\vec{F} = m\vec{a}$ implies the motion is planar.

Problem 62 By the previous problem we find the orbital motion of a particular planet revolving around the sun must lie in a plane since Newton's universal law of gravitation is a central force. To a good approximation we can take the sun as motionless at the center of the solar system. Place the center of the sun at the origin and use xy-coordinates to label the plane of orbital motion. Also, use polar coordinates in the same plane. We can write the force of gravity on a planetat position $\vec{r}$ as follows:

$$\vec{F} = \frac{GmM}{r^2}\widehat{\mathbf{r}} = -\frac{GmM}{r^3}\vec{r}.$$

Show that the angular momentum of the planet is conserved. Recall that the angular momentum of the planet is simply given by $\vec{L} = m\vec{r} \times \vec{v}$. Also, recall that Newton's second law for the planet states $\vec{F} = m\frac{d\vec{v}}{dt}$. You'll need all these facts together with the formula for gravity above to show that $\frac{d\vec{L}}{dt} = 0$.

Problem 63 Suppose two ninja begin travelling the paths given below. To begin, at $t = 0$, a relatively slow genin level ninja sets off in a NE direction given by

$$\vec{r}_1(t) = \langle -10 + t, 1 + t\rangle.$$

However, at the same time $t = 0$, an enemy Jonin sets off in a NW direction given by

$$\vec{r}_2(t) = \langle 20 - 4t, 6 + t\rangle.$$

Both of these paths are placed in a forest thick with a mist which lowers visability to near zero. Suppose the Jonin level ninja has advanced tracking skills that allow his to pick up on the faintest of scents. If he crosses the path of an enemy he can smell it and then alter his path to pursue and attack the enemy genin. Should the genin worry? Is he in danger? (a Jonin is no match for a typical genin in a usual battle, if the Jonin catches the genin it's game over for the lowly genin)

Problem 64 Suppose the velocity is given by $\vec{v}(t) = \langle t, 3, t\cosh(t^2)\rangle$ for some particle which has initial position $(1, 2, 3)$. Find the acceleration and position at time $t \geq 0$.

Problem 65 Suppose $\vec{a} = 3\,\widehat{\mathbf{x}}$. Given this constant acceleration, derive the velocity and position as functions of time. Please state your answers in terms of arbitrary initial position $\vec{r}(0) = \vec{r}_o$ and initial velocity $\vec{v}(0) = \vec{v}_o$.

Problem 66 Suppose $\vec{r}(t) = \langle 2^t, \ln(t), \sqrt{t^2 + 1}\rangle$. Find the parametric equations of the tangent line to the given curve at the point $(8, \ln(8), \sqrt{65})$.

Problem 67 Flashback: calculate: $\int \sin^n(x)\ ,dx$ for $n = 2$ and $n = 3$. (by hand, no Mathematica here please, at least not in your solution)

Problem 68 Flashback: calculate: (simplify answer please, assume $t > 0$)

$$\frac{d}{dt}\left[2t\cosh(2\ln(t))\right]$$

Problem 69 Suppose $\vec{r}(t) = \langle 2^{-t}, 3\sin(t), 4t\rangle$ for $t \geq 0$ describes the path travelled by a hover car. Set-up, but do not evaluate, the distance travelled by time t for this path. Also, if this path was given in units of miles per second and the hover craft police placed a speed limit of 6mps on hover cars then is there any chance an honest hover cop will give you a ticket? Try to estimate a reasonable upper bound on the speed of the given hover car path.

Problem 70 Calculate the limit below if it exists. If it does not exist then show it fails to exist.

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$$

Problem 71 Calculate the limit below if it exists. If it does not exist then show it fails to exist.

$$\lim_{(x,y)\to(B,B)} \frac{x^4-y^4}{x^2-y^2}$$

Problem 72 Determine if the limit below exists. If it does exist, calculate it's value. If it does not exist then give an explicit argument which shows it cannot converge to a real value.

$$\lim_{(x,y)\to(0,0)} \left[\frac{y^2x}{y^4+x^2}\right].$$

Problem 73 Calculate the limit below by switching to polar coordinates.

$$\lim_{(x,y)\to(0,0)} \left[\frac{\sin(x^2+y^2)}{2x^2+2y^2}\right].$$

Problem 74 Show that the limit of

$$f(x,y) = \lim_{(x,y)\to(0,0)} \left[\frac{x-y^2}{x^2+y^2}\right]$$

does not converge to a real number.

Problem 75 Given that f is continuous at the origin, what must the value of A be in the definition below?

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2+y^2} & (x,y) \neq 0 \\ A & (x,y) = (0,0) \end{cases}$$