These problems are worth 1pt a piece at least. Feel free to use Mathematica or some other CAS to illustrate as needed.

Problem 126 Apply the method of Lagrange multipliers to solve the following problem: Let a, b be constants. Maximize xy on the ellipse $b^2x^2 + a^2y^2 = a^2b^2$.

Problem 127 Apply the method of Lagrange multipliers to solve the following problem: Find the distance from $(1, 0)$ to the parabola $x^2 = 4y$.

Problem 128 Apply the method of Lagrange multipliers to solve the following problem: Suppose the base of a rectangular box costs twice as much per square foot as the sides and the top of the box. If the volume of the box must be $12ft^3$ then what dimensions should we build the box to minimize the cost? *[Please state the dimensions of the base and altitude clearly. Include a picture in your solution to explain the meaning of any variables you introduce, thanks!]*

Problem 129 Taking a break from the method of Lagrange. Assume a, b, c are constants: Show that the surfaces $xy = az^2$, $x^2 + y^2 + z^2 = b$ and $z^2 + 2x^2 = c(z^2 + 2y^2)$ are mutually perpendicular.

Problem 130 Apply the method of Lagrange multipliers to derive a formula for the distance from the plane $ax + by + cz + d = 0$ to the origin. If necessary, break into cases.

Problem 131 Suppose you want to design a soda can to contain volume V of soda. If the can must be a right circular cylinder then what radius and height should you use to minimize the cost of producing the can? *assume the cost is directly proportional to the surface area of the can*

Problem 132 Find any extreme values of xy^2z on the sphere $x^2 + y^2 + z^2$. *note the sphere is compact and the function $f(x, y, z) = xy^2z$ is continuous so this problem will have at least two interesting answers*

Problem 133 Again, breaking from optimization, this problem explores a concept some of you have not yet embraced. Find the point(s) on $x^2 + y^2 + z^2 = 4$ which the curve $\vec{r}(t) = \langle \sin(t), \cos(t), t \rangle$ intersects.

Problem 134 Consider $f(x, y) = x^3 - 3x - y^2$. Find any critical points for f and use the second derivative test for functions of two variables to judge if any of the critical points yield local extrema.

Problem 135 Consider $f(x, y) = x^2 - y^2$. Find any critical points for f and use the second derivative test for functions of two variables to judge if any of the critical points yield local extrema.

Problem 136 Consider $f(x, y) = x^3 + y^3 - 3xy$. Find any critical points for f and use the second derivative test for functions of two variables to judge if any of the critical points yield local extrema.

Problem 136+i An armored government agent decides to investigate a disproportionate use of electricity in a gated estate. Foolishly entering without a warrant he find himself at the mercy of Ron Swanson (at $(1,0,0)$), Dwight Schrute (at $(-1,1,0)$) and Kakashi (in a tree at $(1,1,3)$). Supposing Ron Swanson inflicts damage at a rate of 5 units inversely proportional from the square of his distance to the agent, and Dwight inflicts constant damage at a rate of 3 in a sphere of radius 2. If Kakashi inflicts a damage at a rate of 5 units directly proportional to the square of his distance from his location (because if you flee it only gets worse the further you run as he attacks you retreating) then where should you assume a defensive position as you call for back-up? What location minimizes your damage rate? Assume the ground is level and you have no jet-pack and/or antigravity devices.

Problem 137 Find global extrema for $f(x, y) = exp(x^2 - 2x + y^2 - 6y)$ on the closed region bounded by $x^2/4 + y^2/16 = 1$.

Problem 138 Find the maximum and minimum values for $f(x,y) = x^2 + y^2 - 1$ on the region bounded by the triangle with vertices $(-3,0), (1,4)$ and $(0,-3)$.

Problem 139 Find the maximum and minimum values for $f(x, y) = x^4 - 2x^2 + y^2 - 2$ on the closed disk with boundary $x^2 + y^2 = 9$.

Problem 140 Find the multivariate power series expansion for $f(x, y) = ye^x \sin(y)$ centered at $(0, 0)$

Problem 141 Expand $f(x, y, z) = xyz + x^2$ about the center $(1, 0, 3)$.

Problem 142 Given that $f(x, y) = 3 + 2x^2 + 3y^2 - 2xy + \cdots$ determine if $(0, 0)$ is a critical point and is $f(0, 0)$ a local extremum.

Problem 143 Use Clairaut's Theorem to show it is impossible for $\vec{F} = \langle y^3 + x, x^2 + y \rangle = \nabla f$.

Problem 144 Suppose $\vec{F} = \langle P, Q \rangle$ and suppose $P_y = Q_x$ for all points in some subset $U \subseteq \mathbb{R}^2$. Does it follow that $\vec{F} = \nabla f$ on U for some scalar function f? Discuss.
Hint: the polar angle θ has total differential $d\theta = d(\tan^{-1}(y/x)) = \frac{y}{x^2+y^2}dx - \frac{x}{x^2+y^2}dx$, think about the example $\vec{F} = \langle \frac{y}{x^2+y^2}, -\frac{x}{x^2+y^2} \rangle$. This function has domain $U = \mathbb{R}^2 - \{(0,0)\}$, can you find f such that $\vec{F} = \nabla f$ on all of U?

Problem 145 We say $U \subseteq \mathbb{R}^n$ is path-connected iff any pair of points in U can be connected by a polygonal-path (this is a path made from stringing together finitely many line-segments one after the other) . Show that if $\nabla f = 0$ on a path-connected set $U \subseteq \mathbb{R}^n$ then $f(\vec{x}) = c$ for each $\vec{x} \in U$. You may use the theorem from calculus I which states that if $f'(t) = 0$ for all t in a connected domain then $f = c$ on that domain.

Problem 146 Show that if $\nabla f = \nabla g$ on a path-connected set $U \subseteq \mathbb{R}^n$ then $f(\vec{x}) = g(\vec{x}) + c$ for each $\vec{x} \in U$. *Hint: you can use Problem 145.*

Problem 147 Prove the mean-value theorem for functions $\mathbb{R}^n \xrightarrow{f} \mathbb{R}$. In particular, show that if f is differentiable at each point of the line-segment connecting $\vec{P}$ and $\vec{Q}$ then there exists a point $\vec{C}$ on the line-segment $\overline{PQ}$ such that $\nabla f(\vec{C}) \bullet (\vec{Q} - \vec{P}) = f(\vec{Q}) - f(\vec{P})$.
Hint: parametrize the line-segment and construct a function on $\mathbb{R}$ to which you can apply the ordinary mean value theorem, use the multivariate chain-rule and win.

Problem 148 The method of characteristics is one of the many calculational techniques suggested by the total differential. The idea is simply this: given $dx/dt = f(x, y)$ and $dy/dt = g(x, y)$ we can solve both of these for dt to eliminate time. This leaves a differential equation in just the cartesian coordinates x, y and we can usually use a separation of variables argument to solve for the level curves which the solutions to $dx/dt = f(x, y)$ and $dy/dt = g(x, y)$ parametrize. Use the technique just described to solve

$$\frac{dx}{dt} = -y \qquad \& \qquad \frac{dy}{dt} = x.$$

Problem 149 Suppose that the force $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$ is the net-force on a mass m. Furthermore, suppose $\vec{B} = B\,\widehat{\mathbf{z}}$ and $\vec{E} = E\,\widehat{\mathbf{z}}$ where E and B are constants. Find the equations of motion in terms of the initial position $\vec{r}_o = \langle x_o, y_o, z_o \rangle$ and velocity $\vec{v}_o = \langle v_{ox}, v_{oy}, v_{oz} \rangle$ by solving the differential equations given by $\vec{F} = m\frac{d\vec{v}}{dt}$. If $E = 0$ and $v_{oz} = 0$ then find the radius of the circle in which the charge q orbits.
Hint: first solve for the velocity components via the technique from Problem 148 then integrate to get the components of the position vector.

Problem 150 Suppose objective function $f(x, y)$ has an extremum on $g(x, y) = 0$. Show that F defined by $F(x, y, \lambda) = f(x, y) - \lambda g(x, y)$ recovers the extremum as a critical point. From this viewpoint, the adjoining of the multiplier converts the constrained problem in n-dimensions to an unconstrained problem in $(n+1)$-dimensions (you can easily generalize your argument to $n > 2$).