These problems are worth 1pt a piece at least. Feel free to use Mathematica or some other CAS to illustrate as needed.

Problem 151 Calculate

$$\int_0^2 \int_0^4 (3x+4y)\,dx\,dy$$

Problem 152

$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x)\cos(y)\,dx\,dy$$

Problem 153

$$\int_{-1}^{1} \int_0^1 \sin^3(x)\cos^{42}(y)\,dy\,dx$$

Problem 154 Calculate the average of $f(x, y) = x^2 + y^2$ on the unit-square.

Problem 155 Calculate the average of $f(x, y) = x^2 + y^2$ on the region bounded by $x^2 + y^2 = R^2$.

Problem 156 Calculate the average of $f(x, y) = xy$ on $[1, 2] \times [3, 4]$.

Problem 157 Show that

$$\lim_{n\to\infty} \int_0^1 \int_0^1 x^n\, y^n\, dx\, dy = 0.$$

Problem 158 Calculate

$$\int_0^{\ln(2)} \int_0^{\ln(3)} e^{x+y} dx\, dy$$

Problem 159 Suppose $\int\int_R f\, dA = \int_0^1 \int_{x^2}^{\sqrt{x}} (1+x) dy\, dx$. Calculate the given integral.

Problem 160 For the integral given in the previous problem, explicitly write R as a subset of $\mathbb{R}^2$ using set-builder notation. In addition, calculate the integral once more with the interation of the integrals beginning with dx. Draw a picture to explain the inequalities which form the basis for your new set-up to the integral.

Problem 161 Reverse the order of integration in order to calculate the following integral:

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2)\, dx\, dy.$$

Problem 162 Reverse the order of integration in order to calculate the following integral:

$$\int_0^1 \int_y^1 \frac{1}{1+x^4}\, dx\, dy.$$

Problem 163 Find the average of $f(x, y) = xy$ over the triangle with vertices $(0, 0)$, $(3, 1)$ and $(-2, 4)$.

Problem 164 Find volume bounded by $z = y + e^x$ and the xy-plane for $(x, y) \in [0, 1] \times [0, 2]$.

Problem 165 Find the volume bounded inside the cylinder $x^2 + y^2 = 1$ and the planes $z = x + 1$ and $z = y - 3$.

Problem 166 Find the volume bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

Problem 167 Calculate the integral (use polars):

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2)^{3/2}\, dy\, dx.$$

Problem 168 Calculate the integral (use polars):

$$\int_0^1 \int_x^1 (x^2+y^2)^3\, dy\, dx.$$

Problem 169 Suppose R is the region bounded by $y + |x|$ and $x^2 + (y-1)^2 = 1$. Express R in polar coordinates. In other words, draw a picture and indicate how the points in R are reached by particular ranges of r and θ.

Problem 170 Find volume bounded by the paraboloid $x = y^2 + 2z^2$ and the parabolic cylinder $x = 2 - y^2$.

Problem 171 Find the volume bounded by the cones $z = \sqrt{x^2 + y^2}$ and $z = 2\sqrt{x^2 + y^2}$ and the sphere $\rho = 3$.

Problem 172 Let B be a ball of radius R centered at the origin. Calculate $\int\int\int_B e^{-\rho^3}\, dV$

Problem 173 Let $u = \frac{2x}{x^2+y^2}$ and $v = \frac{-2y}{x^2+y^2}$ calculate $\frac{\partial(x,y)}{\partial(u,v)}$.

Problem 174 Suppose $\delta(x, y, z) = 1 = dM/dV$ for $x, y, z > 0$. Find center of mass for a sphere with this density δ centered at $(1, 2, 3)$.

Problem 175 Suppose $\delta(x, y, z) = xyz = dM/dV$ for $x, y, z > 0$. Find center of mass for a sphere with this density centered at $(1, 2, 3)$.