These problems are worth 1pt a piece at least. Feel free to use Mathematica or some other CAS to illustrate as needed.

Problem 175+i Suppose you have a cylindrical oil tank which is placed on a hill close to your house. After some time the land settles and the oil tank is not level. Suppose you read a dip-stick which is designed for a level-set-up and find the tank is half-full. Suppose the tank is slanted at 20 degrees relative to the true horizontal. In other words, suppose the axis of the cylinder makes and angle of 70 degrees with the vertical. Fortunately, the tank is only tilted along that direction and the perpendicular direction to the central-axis remains at a right angle to the vertical. If you have a 1000gallon tank then how much oil do you really have?
numerical integration is totally fine here, although, this may have a closed-form solution.

Problem 176 Calculate $\int_R \sqrt{x+2y}\sin(x-y)\,dA$ where $R=[0,1]\times[0,1]$ by making an appropriate change of variables.

Problem 177 Find the center of mass for a laminate of variable density $\delta(r,\theta) = r\sin(\theta)$ which is bounded by $r=\sin(2\theta)$

Problem 178 Let $\vec{F}(x,y,z) = \langle x+z, x+y^2, z^3\rangle$ calculate $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$.

Problem 179 Suppose a,b,c are constants. Let $\vec{G}(x,y,z) = \langle x+z+a, x+y^2+b, z^3+c\rangle$ calculate $\nabla\cdot\vec{G}$ and $\nabla\times\vec{G}$.

Problem 180 Find a function f and a vector field $\vec{A}$ such that $\vec{F} = \nabla f + \nabla\times\vec{A}$ where $\vec{F}$ is the vector field studied in problem 178.

Problem 181 Suppose $\nabla\times\vec{F} = \nabla\times\vec{G}$. Does it follow that $\vec{F}=\vec{G}$?

Problem 182 Suppose $\nabla\cdot\vec{F} = \nabla\cdot\vec{G}$. Does it follow that $\vec{F}=\vec{G}$?

Problem 183 Suppose $\nabla\times\vec{F} = \nabla\times\vec{G}$ and $\nabla\cdot\vec{F} = \nabla\cdot\vec{G}$ make a conjecture: does it follow that $\vec{F}=\vec{G}+\vec{c}$ for some constant vector $\vec{c}$? (no work required if your answer is yes, however if your answer is no then I would like for you to provide a counter-example)

Problem 184 Show that

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B}).$$

Problem 185 Given $\nabla\cdot\vec{E} = \rho/\epsilon_o$ and $\nabla\times\vec{B} = \mu_o\vec{J} + \mu_o\epsilon_o\frac{\partial\vec{E}}{\partial t}$ show that $\nabla\cdot\vec{J} = \frac{\partial\rho}{\partial t}$. If $\vec{J}$ is the charge per unit area time following in the area with direction $\widehat{J}$ and ρ is the charge per unit volume then what does this equation mean physically speaking?

Problem 186 Calculate $\nabla(\vec{A}\cdot(\vec{B}\times\vec{C}))$ where $\vec{A},\vec{B},\vec{C}$ are all smooth vector fields.

Problem 187 $\displaystyle\int_C 3x^2yz\,ds$ where C is the curve parameterized by $\mathbf{X}(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ and $0\le t\le 1$.

Problem 188 Find the centroid of the curve C: the upper-half of the unit circle plus the x-axis from -1 to 1.
Hint: Use geometry and symmetry to compute 2 of the 3 line integrals.

Problem 189 Find the centroid of the helix C parameterized by $\mathbf{X}(t) = (2\sin(t), 2\cos(t), 3t)$ where $0 \leq t \leq 2\pi$.

Problem 190 Let $\vec{F} = \langle z, y, x \rangle$. Calculate $\int_C \vec{F} \bullet d\vec{r}$ for the line-segment C from $(1,1,1)$ to $(3,4,5)$.

Problem 191 Let $\vec{F} = \langle 0, 0, -mg \rangle$ where m, g are positive constants. Find the work done as you travel up the helix $\vec{r}(t) = \langle R\cos(t), R\sin(t), t \rangle$ for $0 \leq t \leq 4\pi$.

Problem 192 Let $\vec{F} = \langle 0, 0, -mg \rangle$ where m, g are positive constants and suppsoe $\vec{F}_f = -b\vec{T}$ where v is your speed and b is a constant and $\vec{T}$ is the unit-vector which points along the tangential direction of the path. This is a simple model of the force of kinetic friction, it just acts opposite your motion. Find the work done by $\vec{F}_f + \vec{F}$ as you travel up the helix $\vec{r}(t) = \langle R\cos(t), R\sin(t), t \rangle$ for $0 \leq t \leq 4\pi$.

Problem 193 Let $\vec{F}(x,y) = \langle 1 + y, -x + 2 \rangle$. Let C be the ellipse $x^2/4 + y^2/9 = 1$ given a CCW orientation. Calculate $\int \vec{F} \bullet d\vec{r}$.

Problem 194 Let $\mathbf{F}(x,y) = (P(x,y), Q(x,y))$ be a vector field where
$P_y = Q_x$ except at the points P_1, P_2, and P_3.
Suppose that $\int_{C_1} \mathbf{F} \cdot d\mathbf{X} = 1$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{X} = 2$, $\int_{C_3} \mathbf{F} \cdot d\mathbf{X} = 3$, and $\int_{C_4} \mathbf{F} \cdot d\mathbf{X} = 4$ where C_1, C_2, $\ldots C_8$ are pictured in the supplement to Problem Set 8.

Compute:

(a.) $\int_{C_5} \mathbf{F} \cdot d\mathbf{X}$ [Answer: 6]

(b.) $\int_{C_6} \mathbf{F} \cdot d\mathbf{X}$

(c.) $\int_{C_7} \mathbf{F} \cdot d\mathbf{X}$

(d.) $\int_{C_8} \mathbf{F} \cdot d\mathbf{X}$

Problem 195 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\mathbf{F}(x,y) = (x^2 + y^2, xy)$
(b.) $\mathbf{F}(x,y) = (2x + 3x^2y^2 + 5, 2x^3y)$
(c.) $\mathbf{F}(x,y) = \left(e^x, ye^{-y^2}\right)$

Problem 196 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\mathbf{F}(x,y) = (e^{xy}, x^4y^3 + y)$

(b.) $\mathbf{F}(x,y) = \left(e^x + \dfrac{y}{1+x^2}, \arctan(x) + (1+y)e^y\right)$

(c.) $\mathbf{F}(x,y,z) = (yz + y + 1, xz + x + z, xy + y + 1)$

Problem 197 Let $\vec{E}(x,y,z) = \frac{1}{(x^2+y^2+z^2)^{(3/2)}}\langle x,y,z\rangle$. Calculate $\nabla \bullet \vec{E}$ and $\nabla \times \vec{E}$. (use cartesians or spherical coordinates, your choice (the formulas for divergence and curl in sphericals are contained within the pdf on "VectorFieldDifferentiation" in course content). Plot this vector field (don't have to turn in plot, I trust you to do it), are your calculations surprising?

Problem 198 Let C be the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ (oriented counter-clockwise). Compute the line integral: $\int_C y^2\,dx + x^2\,dy$ two ways. First, compute the integral directly by parameterizing each side of the square. Then, compute the answer again using Green's Theorem.

Problem 199 Compute $\int_C \mathbf{F} \cdot d\mathbf{X}$ where $\mathbf{F}(x,y) = (y - \ln(x^2+y^2), 2\arctan(y/x))$ and C is the circle $(x-2)^2 + (y-3)^2 = 1$ oriented counter-clockwise.

Problem 200 prove the other half of Green's theorem. (I only proved half in lecture).