remember, you can check your work with a CAS, but it is you who must do the work. Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations.

Problem 1 The Cartesian product of three sets A, B and C is defined as follows:

$$A \times B \times C = \{(a, b, c) \mid a \in A,\ b \in B,\ c \in C\}.$$

If $(x, y, z) \in [0, 1] \times \mathbb{R} \times \mathbb{Z}$ then what can you tell me about x, y and z?

Problem 2 Suppose $P = (1, 2)$, $Q = (-1, -2)$ and $R = (0, 3)$. What point is in the middle of these three points?
Hint: the "middle" is found by the vector-average of these points; the middle point M is simply given by $M = \frac{1}{3}(P + Q + R)$. This idea also works for 4,5 or 500 points.

Problem 3 * Find the angle between the lines which connect the center of a tetrahedron and two vertices of a unit tetrahedron via a vector argument. (chemistry sees this shape in CH_4)

Problem 4 The displacement from point P to point Q is defined to be the vector which points from P to Q, in particular $\overrightarrow{PQ} = Q - P$. Suppose $A = (1, 2, 3)$, $B = (1, 1, -2)$ and $C = (4, 4, 4)$.

(a) calculate the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$.

(b) calculate $\overrightarrow{AB} + \overrightarrow{BC}$

(c) calculate $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

(d) find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$, call it θ

(e) find the angle between $\overrightarrow{CA}$ and $\overrightarrow{CB}$, call it β

(f) find the angle between $\overrightarrow{BC}$ and $\overrightarrow{BA}$, call it α

(g) calculate $\theta + \beta + \alpha$. Does your result make sense? Comment on the geometric meaning of this problem.

Problem 5 Are the vectors $\vec{v} = \langle 1, 0, 4\rangle$ and $\vec{w} = \langle 0, 2, 0\rangle$ orthogonal, colinear or neither? (give a calculation and explain)

Problem 6 Calculate the projection of $\vec{v} = \langle 1, 1, 1\rangle$ onto the vector $\vec{w} = 2\,\widehat{y} - \widehat{z}$. Write $\vec{v}$ as a sum of a vector which is colinear to $\vec{w}$ and another vector which is orthogonal to $\vec{w}$.

Problem 7 Suppose $\vec{A} = \widehat{x} + \widehat{y}$ and $\vec{B} = \widehat{z}$ and $\vec{C} = \widehat{y}$.

(a) calculate $\vec{A} \bullet (\vec{B} \times \vec{C})$

(b) calculate $\vec{B} \bullet (\vec{A} \times \vec{C})$

(c) which of the calculations above gives the volume of the parallel piped spanned by the given vectors ?

Problem 8 Suppose $\vec{A}, \vec{B}$ and $\vec{C}$ are vectors which do not all lie in a common plane. When does $\vec{A} \bullet (\vec{B} \times \vec{C})$ yield the volume of the parallel piped with edges $\vec{A}, \vec{B}$ and $\vec{C}$?

Problem 9 Find the equation of a plane which contains the line parametrized by $\vec{r}(t) = \langle 1+t, 2-t, 3\rangle$ and the vector $\vec{w} = \langle 1, 2, 3\rangle$.

Problem 10 (this problem is bonus!) I explain in the notes that a convenient compact notation for the cross-product is provided by the antisymmetric symbol ϵ_{ijk} in particular:

$$\vec{A} \times \vec{B} = \sum_{i,j,k=1}^{3} \epsilon_{ijk} A_i B_j \,\hat{x}_k.$$

The identity below can be checked case-by-case, it is true for all m, n, i, k in $\{1, 2, 3\}$,

$$\boxed{\sum_{j=1}^{3} \epsilon_{mnj}\epsilon_{ikj} = \delta_{mi}\delta_{nk} - \delta_{mk}\delta_{ni}. \quad \star}$$

I give you two options: (choose just one please)

(a.) show the $\star$ identity is true for all $3^4 = 81$ cases.

(b.) show that $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \bullet \vec{C})\vec{B} - (\vec{A} \bullet \vec{B})\vec{C}$ by implementing the $\star$-identity in an index-calculation.

Problem 11 Suppose a force $\vec{F}$ with magnitude of 100 in the $\hat{x}$-direction is applied to a mass displacing it from the point $(1, 2, 3)$ to the point $(4, 4, 4)$. Calculate the work done by $\vec{F}$ (assume $\vec{F}$ is constant).

Problem 12 Suppose a rigid body rotates with angular frequency $\vec{\omega}$. It is customary for the direction of $\vec{\omega}$ to point along the axis of rotation while the magnitude ω gives the number of radians rotated per time period. It is known that the velocity of a particle on such a rigid body at $\vec{r}$ from the axis of rotation is simply given by $\vec{v} = \vec{\omega} \times \vec{r}$. Here the vector $\vec{r}$ points from the axis to the point on the body which has velocity $\vec{v}$. See the picture below, the choice of $\vec{r}$ is not unique. Why is the formula $\vec{v} = \vec{\omega} \times \vec{r}$ unambiguous? Why is $\vec{\omega} \times \vec{r}_1 = \vec{\omega} \times \vec{r}_2$ where $\vec{r}_1, \vec{r}_2$ are related as indicated in the diagram below? (please remind me to draw this in class, hopefully this will allow all involved to get the right picture in mind)

Problem 13 Find the direction vector of the line of intersection of the planes $x + y + z = 3$ and $2x - 3y - 4z = 7$.

Problem 14 Explain what the parametric equations $x = u + v, y = u - v$ and $z = 1 + u$ describe. Find the corresponding cartesian equations (either give answer as a graph or as a level surface, please say which your answer is)

Problem 15 Suppose a plane S contains the points $(1, 0, 2), (3, 4, 1)$ and $(0, 0, 1)$. Find the point on S which is closest to the origin.

Problem 16 Refer once more to the plane S from the previous problem. Find a parametrization of the triangular region bounded by the points given in the previous problem.

Problem 17 Suppose $\vec{a} \bullet \vec{b} = \vec{a} \bullet \vec{c}$ for some vector $\vec{a}$. Does it follow that $\vec{b} = \vec{c}$?

Problem 18 Suppose $\vec{a} \bullet \vec{b} = \vec{a} \bullet \vec{c}$ for **all** vectors $\vec{a}$. Does it follow that $\vec{b} = \vec{c}$?

Problem 19 Suppose $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ for some vector $\vec{a}$. Does it follow that $\vec{b} = \vec{c}$?

Problem 20 Suppose $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ for **all** vectors $\vec{a}$. Does it follow that $\vec{b} = \vec{c}$?

Problem 21 * **Claim:** If $\vec{v} \bullet \vec{x} = c$ and $\vec{v} \times \vec{x} = \vec{b}$ for particular nonzero vectors $\vec{v}, \vec{b}$ and some constant c then we can solve for $\vec{x}$ uniquely in terms of the given vectors $\vec{v}, \vec{b}$ and c. **Show the claim is true**.

Problem 22 Suppose that $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are coplanar vectors in $\mathbb{R}^3$. Show that $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D}) = 0$.

Problem 23 Use vectors to show that the diagonals of a parallelogram are orthogonal iff the parallelogram is a rhombus.

Problem 24 * Suppose that A, B, C, D are points in $\mathbb{R}^3$ where no three of these points lie on the same line. Furthermore, suppose S is the quadrilateral $ABCD$ formed from these points. Note S need not lie in a plane. Let M_1, M_2, M_3, M_4 be the midpoints of of the edges of S. Use a vector argument to show $M_1M_2M_3M_4$ is a parallelogram!

Problem 25 This problem explores the concept of **direction cosines**, these gadgets are still popular in certain applications, we don't use them much in this course, but here it is for breadth and also practice on your dot-product skilz. Find the **cosine of the angle** between $\vec{v} = \langle 2, 4, \sqrt{5} \rangle$ and the

(a) positive x-axis (usually denoted $\cos(\alpha)$)

(b) positive y-axis (usually denoted $\cos(\beta)$)

(c) positive z-axis (usually denoted $\cos(\gamma)$)

(d) how are the answers to parts a, b, c related to $\vec{v}$?