

Feel free to use Mathematica or some other CAS to illustrate as needed.

Problem 178 Let $\vec{F}(x, y, z) = \langle x+z, x+y^2, z^3 \rangle$ calculate $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$.

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x+z) + \frac{\partial}{\partial y}(x+y^2) + \frac{\partial}{\partial z}(z^3) = \boxed{1+2y+3z^2}$$

$$\begin{aligned}\nabla \times \vec{F} &= \left\langle \frac{\partial}{\partial y}(z^3) - \frac{\partial}{\partial z}(x+y^2), \frac{\partial}{\partial z}(x+z) - \frac{\partial}{\partial x}(z^3), \frac{\partial}{\partial x}(x+y^2) - \frac{\partial}{\partial y}(x+z) \right\rangle \\ &= \boxed{\langle 0, 1, 1 \rangle}\end{aligned}$$

Problem 179 Suppose a, b, c are constants. Let $\vec{G}(x, y, z) = \langle x+z+a, x+y^2+b, z^3+c \rangle$ calculate $\nabla \cdot \vec{G}$ and $\nabla \times \vec{G}$.

$$\nabla \cdot \vec{G} = \frac{\partial}{\partial x}(x+z+a) + \frac{\partial}{\partial y}(x+y^2+b) + \frac{\partial}{\partial z}(z^3+c) = \boxed{1+2y+3z^2}$$

$$\begin{aligned}\nabla \times \vec{G} &= \left\langle \frac{\partial}{\partial y}(z^3+c) - \frac{\partial}{\partial z}(x+y^2+b), \frac{\partial}{\partial z}(x+z+a) - \frac{\partial}{\partial x}(z^3+c), \frac{\partial}{\partial x}(x+y^2+b) - \frac{\partial}{\partial y}(x+z+a) \right\rangle \\ &= \boxed{\langle 0, 1, 1 \rangle}\end{aligned}$$

Problem 180 Find a function f and a vector field \vec{A} such that $\vec{F} = \nabla f + \nabla \times \vec{A}$ where \vec{F} is the vector field studied in problem 178.

$$\vec{F} = \langle x+z, x+y^2, z^3 \rangle$$

$$\vec{F} = \underbrace{\langle x, y^2, z^3 \rangle}_{\begin{array}{c} \text{gave terms} \\ \text{in } \nabla \cdot \vec{F} \end{array}} + \underbrace{\langle z, x, 0 \rangle}_{\begin{array}{c} \text{gave terms} \\ \text{in } \nabla \times \vec{F} \end{array}}$$

We can use

$$f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{3}y^3 + \frac{1}{4}z^4$$

Find \vec{A} such that

$$\nabla \times \vec{A} = \underbrace{\langle z, x, 0 \rangle}_{\text{focus here}}$$

We need $\vec{A} = \langle A_1, A_2, A_3 \rangle$ such that,

$$(*) \begin{cases} \partial_y A_3 - \partial_z A_2 = z & (\text{I.}) \\ \partial_z A_1 - \partial_x A_3 = x & (\text{II.}) \\ \partial_x A_2 - \partial_y A_1 = 0 & (\text{III.}) \end{cases}$$

I'll use guessing to solve, make a proposal, see if it is flexible enough to solve $(*)$

Let $A_2 = -z^2/2$ then (I.) is solved. (if $\partial_z A_2 = 0$)

Let $A_3 = -x^2/2$ then (II.) is solved (if $\partial_x A_3 = 0$)

Let $A_1 = 0$ then III is solved.

Let's see $\vec{A} = \langle 0, -\frac{1}{2}z^2, -\frac{1}{2}x^2 \rangle$ then

$\nabla \times \vec{A} = \langle z, x, 0 \rangle$ as needed and we're done.

$$\boxed{\vec{F} = \nabla \left(\frac{1}{2}x^2 + \frac{1}{3}y^3 + \frac{1}{4}z^4 \right) + \nabla \times \langle 0, -\frac{1}{2}z^2, -\frac{1}{2}x^2 \rangle}$$

Problem 181 Suppose $\nabla \times \vec{F} = \nabla \times \vec{G}$. Does it follow that $\vec{F} = \vec{G}$?

It does not follow since $\nabla \times (\nabla f) = 0$ we can have $\vec{F} - \vec{G} = \nabla f$ and both \vec{F} & \vec{G} will have same curl and yet $\vec{F} \neq \vec{G}$. For example,

$$\vec{F} = \langle x, y, z \rangle, \quad \vec{G} = \langle x, y, z \rangle + \langle 1, 1, 1 \rangle$$

Both have $\nabla \times \vec{F} = 0 = \nabla \times \vec{G}$ and clearly $\vec{F} \neq \vec{G}$.

Problem 182 Suppose $\nabla \cdot \vec{F} = \nabla \cdot \vec{G}$. Does it follow that $\vec{F} = \vec{G}$?

Notice $\nabla \cdot (\nabla \times \vec{A}) = 0$ thus adding a $\nabla \times \vec{A}$ term does not alter the divergence; $\nabla \cdot \vec{F} = \nabla \cdot (\vec{F} + \nabla \times \vec{A})$.

Of course, both 181 & 182 are also easily solved by noting div. and curl send constant vectors to zero. Could make example, $\vec{F}, \vec{G} = \vec{F} + \vec{c}$

Problem 183 Suppose $\nabla \times \vec{F} = \nabla \times \vec{G}$ and $\nabla \cdot \vec{F} = \nabla \cdot \vec{G}$ make a conjecture: does it follow that $\vec{F} = \vec{G} + \vec{c}$ for some constant vector \vec{c} ? (no work required if your answer is yes, however if your answer is no then I would like for you to provide a counter-example)

I suspect the conjecture is false. Let's focus on z -components and see what we need:

$$\partial_x F_y - \partial_y F_x = \partial_x G_y - \partial_y G_x$$

$$\partial_x F_x + \partial_y F_y + \partial_z F_z = \partial_x G_x + \partial_y G_y + \partial_z G_z$$

Can I solve these while $\vec{F} \neq \vec{G}$? (I'll aim for $\nabla \times \vec{F}, \nabla \times \vec{G}$ zero in x, y components)

$$F_x = x - y, \quad F_y = y + x$$

$$G_x = x, \quad G_y = y$$

$$\text{We find } \partial_x F_y - \partial_y F_x = 1 - 1 \neq 0$$

$$\partial_x G_y - \partial_y G_x = 0 - 0 \neq 0$$

Set $G_z = F_z = 0$ and observe $\nabla \cdot \vec{F} = 2 = \nabla \cdot \vec{G}$

Hence $\vec{F} = \langle x - y, x + y, 0 \rangle$ & $\vec{G} = \langle x, y, 0 \rangle$ provide a counter-example. There are many others.

Problem 184 Show that

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B}).$$

$$\begin{aligned}
\nabla \cdot (\vec{A} \times \vec{B}) &= \sum_{k=1}^3 \partial_k (\vec{A} \times \vec{B})_k \\
&= \sum_{k=1}^3 \partial_k \left(\sum_{i,j=1}^3 \epsilon_{ijk} A_i B_j \right) \\
&= \sum_{i,j,k} \epsilon_{ijk} \partial_k (A_i B_j) \\
&= \sum_{i,j,k} \epsilon_{ijk} [(\partial_k A_i) B_j + A_i (\partial_k B_j)] \\
&= \sum_{j=1}^3 B_j \left(\sum_{i,k} \epsilon_{kij} \partial_k A_i \right) - \sum_{i=1}^3 A_i \left(\sum_{j,k} \epsilon_{kji} \partial_k B_j \right) \\
&= \sum_{j=1}^3 B_j (\nabla \times \vec{A})_j - \sum_{i=1}^3 A_i (\nabla \times \vec{B})_i \\
&= \underline{\vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})} //
\end{aligned}$$

Or, a page or two brute-force.

Problem 185 Given $\nabla \cdot \vec{E} = \rho/\epsilon_0$ and $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ show that $\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}$. If \vec{J} is the charge per unit area time following in the area with direction \vec{J} and ρ is the charge per unit volume then what does this equation mean physically speaking?

$$\begin{aligned}\nabla \cdot (\nabla \times \vec{B}) &= \nabla \cdot \left(\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \\ \Rightarrow 0 &= \mu_0 \left(\nabla \cdot \vec{J} + \frac{\partial}{\partial t} [\epsilon_0 \nabla \cdot \vec{E}] \right) \\ \Rightarrow \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} &= 0 \\ \therefore \nabla \cdot \vec{J} &= - \frac{\partial \rho}{\partial t} \\ &\qquad\qquad\qquad //\end{aligned}$$

Problem 186 Calculate $\nabla(\vec{A} \cdot (\vec{B} \times \vec{C}))$ where $\vec{A}, \vec{B}, \vec{C}$ are all smooth vector fields.

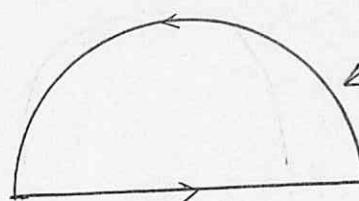
$$\begin{aligned}\nabla(\vec{A} \cdot (\vec{B} \times \vec{C})) &= \sum_k \partial_k (\vec{A} \cdot (\vec{B} \times \vec{C})) \hat{x}_k \\ &= \sum_k [(\partial_k \vec{A}) \cdot (\vec{B} \times \vec{C}) + \vec{A} \cdot \partial_k (\vec{B} \times \vec{C})] \hat{x}_k \\ &= \sum_k [(\partial_k \vec{A}) \cdot (\vec{B} \times \vec{C}) + \vec{A} \cdot \{(\partial_k \vec{B}) \times \vec{C} + \vec{B} \times (\partial_k \vec{C})\}] \hat{x}_k \\ &= \left\langle (\partial_x \vec{A}) \cdot (\vec{B} \times \vec{C}) + \vec{A} \cdot [(\partial_x \vec{B}) \times \vec{C} + \vec{B} \times (\partial_x \vec{C})], \right. \\ &\quad (\partial_y \vec{A}) \cdot (\vec{B} \times \vec{C}) + \vec{A} \cdot [(\partial_y \vec{B}) \times \vec{C} + \vec{B} \times (\partial_y \vec{C})], \\ &\quad \left. (\partial_z \vec{A}) \cdot (\vec{B} \times \vec{C}) + \vec{A} \cdot [(\partial_z \vec{B}) \times \vec{C} + \vec{B} \times (\partial_z \vec{C})] \right\rangle\end{aligned}$$

Problem 187 $\int_C 3x^2yz \, ds$ where C is the curve parameterized by $\mathbf{X}(t) = \left(t, t^2, \frac{2}{3}t^3 \right)$ and $0 \leq t \leq 1$.

$$\begin{aligned}\mathbf{X}'(t) &= \langle 1, 2t, 2t^2 \rangle \Rightarrow \frac{ds}{dt} = \|\mathbf{X}'(t)\| = \sqrt{1 + 4t^2 + 4t^4} \\ \text{Thus, } \int_C 3x^2yz \, ds &= \int_0^1 3(2t)(2t^2) \sqrt{1 + 4t^2 + 4t^4} \, dt \\ &= 12 \int_0^1 t^3 \sqrt{(1+2t^2)^2} \, dt \\ &= 12 \int_0^1 t^3 (1+2t^2) \, dt \\ &= 12 \int_0^1 (t^3 + 2t^5) \, dt \\ &= 12 \left(\frac{1}{4} + \frac{2}{6} \right) \\ &= \boxed{7}\end{aligned}$$

Problem 188 Find the centroid of the curve C : the upper-half of the unit circle plus the x -axis from -1 to 1 .

Hint: Use geometry and symmetry to compute 2 of the 3 line integrals.



$$\begin{aligned}x &= \cos \theta \Rightarrow dx = -\sin \theta d\theta \\ y &= \sin \theta \Rightarrow dy = \cos \theta d\theta \\ 0 \leq \theta \leq \pi &\quad \left. \begin{array}{l} ds = \sqrt{dx^2 + dy^2} \\ \Rightarrow ds = d\theta \end{array} \right\} \end{aligned}$$

$$\begin{aligned}x &= t \\ y &= 0 \quad \left. \begin{array}{l} ds = dt \end{array} \right\} \end{aligned}$$

$$M = \ell(C) = \int_C ds = \int_{-1}^1 dt + \int_0^\pi d\theta = \underline{2+\pi} \quad \leftarrow \text{arc length.}$$

could have argued w/o integration this is clear.

$$x_{cm} = \frac{1}{M} \int_C x \, ds = 0 \quad \text{By symmetry.}$$

$$\begin{aligned}y_{cm} &= \frac{1}{M} \int_C y \, ds = \frac{1}{M} \left(\int_{-1}^1 (0) \, dt + \int_0^\pi \sin \theta \, d\theta \right) \\ &= \frac{1}{2+\pi} \left(-\cos \theta \Big|_0^\pi \right) \\ &= \frac{1}{2+\pi} (-\cos \pi + \cos 0) \\ &= \frac{2}{2+\pi} \end{aligned}$$

Centroid of C .

$(0, \frac{2}{2+\pi})$

Problem 189 Find the centroid of the helix C parameterized by $\mathbf{X}(t) = (2 \sin(t), 2 \cos(t), 3t)$ where $0 \leq t \leq 2\pi$.

Observe $\Sigma'(t) = \langle 2 \cos t, -2 \sin t, 3 \rangle$

$$\frac{ds}{dt} = \|\Sigma'(t)\| = \sqrt{4 \cos^2 t + 4 \sin^2 t + 9} = \sqrt{13}$$

$$M = \int_C ds = \int_0^{2\pi} \sqrt{13} dt = \frac{2\pi\sqrt{13}}{1}$$

(mass for
unit-density
area or length
of C)

By symmetry, $x_{cm} = y_{cm} = 0$.

Consider,

$$z_{cm} = \frac{1}{M} \int_C z ds$$

$$= \frac{1}{2\pi\sqrt{13}} \int_0^{2\pi} 3t \sqrt{13} dt$$

$$= \frac{3\sqrt{13}}{2\pi\sqrt{13}} \left. \frac{t^2}{2} \right|_0^{2\pi}$$

$$= \frac{3}{4\pi} (4\pi^2)$$

$$= \underline{\underline{3\pi}}$$

(0, 0, 3π) is
the centroid of
the helix.

(makes sense this is
half-way up between
 $(0, 2, 0)$ and $(0, 2, 6\pi)$.
in the middle)

Problem 190 Let $\vec{F} = \langle z, y, x \rangle$. Calculate $\int_C \vec{F} \cdot d\vec{r}$ for the line-segment C from $(1, 1, 1)$ to $(3, 4, 5)$.

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_C 3dx + ydy + xdz \\ &= \int_0^1 [(1+4t)(2dt) + (1+3t)(3dt) + (1+2t)(4dt)] \\ &= \int_0^1 [9 + 25t] dt \\ &= 9 + \frac{25}{2} \\ &= \boxed{\frac{43}{2}}\end{aligned}$$

$$\left. \begin{array}{l} x = 1 + 2t \\ y = 1 + 3t \\ z = 1 + 4t \end{array} \right\} 0 \leq t \leq 1$$

Problem 191 Let $\vec{F} = \langle 0, 0, -mg \rangle$ where m, g are positive constants. Find the work done as you travel up the helix $\vec{r}(t) = \langle R \cos(t), R \sin(t), t \rangle$ for $0 \leq t \leq 4\pi$.

$$\vec{F} = \nabla \varphi \quad \text{for } \varphi(x, y, z) = -mgz$$

helix has terminal pts $(R, 0, 0)$ and $(R, 0, 4\pi)$

Apply FTC for line-integrals, no need to calculate as in 190.

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_{(R, 0, 0)}^{(R, 0, 4\pi)} \nabla \varphi \cdot d\vec{r} = \varphi(R, 0, 4\pi) - \varphi(R, 0, 0) \\ &= -mg(4\pi) - mg(0) \\ &= \boxed{-4\pi mg}\end{aligned}$$

Problem 192 Let $\vec{F} = \langle 0, 0, -mg \rangle$ where m, g are positive constants and suppose $\vec{F}_f = -b\vec{T}$ where v is your speed and b is a constant and \vec{T} is the unit-vector which points along the tangential direction of the path. This is a simple model of the force of kinetic friction, it just acts opposite your motion. Find the work done by $\vec{F}_f + \vec{F}$ as you travel up the helix $\vec{r}(t) = \langle R \cos(t), R \sin(t), t \rangle$ for $0 \leq t \leq 4\pi$.

$$\begin{aligned}\int_C (\vec{F} + \vec{F}_f) \cdot d\vec{r} &= \int_C \vec{F} \cdot d\vec{r} + \int_C (\vec{F}_f \cdot \vec{T}) ds \\ &= -4\pi mg + \int_C (-b\vec{T} \cdot \vec{T}) ds \\ &= -4\pi mg - b \int_C ds \\ &= -4\pi mg - b \int_0^{4\pi} \sqrt{R^2 + 1} dt \\ &= \boxed{-4\pi mg - 4\pi b \sqrt{R^2 + 1}}$$

$$\begin{aligned}\frac{d\vec{r}}{dt} &= \langle -R \sin t, R \cos t, 1 \rangle \\ \left\| \frac{d\vec{r}}{dt} \right\| &= \sqrt{R^2 + 1} = \frac{ds}{dt}\end{aligned}$$

Problem 193 Let $\vec{F}(x, y) = \langle 1+y, -x+2 \rangle$. Let C be the ellipse $x^2/4 + y^2/9 = 1$ given a CCW orientation. Calculate $\int_C \vec{F} \cdot d\vec{r}$.

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_0^{2\pi} \langle 1+y, 2-x \rangle \cdot \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle dt \\ &= \int_0^{2\pi} \langle 1+3\sin t, 2-2\cos t \rangle \cdot \langle -2\sin t, 3\cos t \rangle dt \\ &= \int_0^{2\pi} (-2\sin t - 6\sin^2 t + 6\cos t - 6\cos^2 t) dt \\ &= \int_0^{2\pi} (-6) dt \\ &= \boxed{-12\pi}\end{aligned}$$

$$\begin{aligned}x &= 2\cos t \\ y &= 3\sin t\end{aligned}$$

Problem 194 Let $\mathbf{F}(x, y) = (P(x, y), Q(x, y))$ be a vector field where

$P_y = Q_x$ except at the points P_1, P_2 , and P_3 .

Suppose that $\int_{C_1} \mathbf{F} \cdot d\mathbf{X} = 1$, $\int_{C_2} \mathbf{F} \cdot d\mathbf{X} = 2$, $\int_{C_3} \mathbf{F} \cdot d\mathbf{X} = 3$, and $\int_{C_4} \mathbf{F} \cdot d\mathbf{X} = 4$ where C_1, C_2, \dots, C_8 are pictured in the supplement (see Blackboard).

Compute:

(a.) $\int_{C_5} \mathbf{F} \cdot d\mathbf{X}$ [Answer: 6] = 2(3)

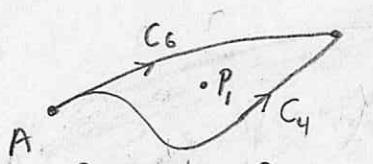
(b.) $\int_{C_6} \mathbf{F} \cdot d\mathbf{X} = 3$ notice P_3 encircled CCW twice.

(c.) $\int_{C_7} \mathbf{F} \cdot d\mathbf{X} = 0$

(d.) $\int_{C_8} \mathbf{F} \cdot d\mathbf{X} = 0$

Need to use Greene's Thm with holes & or reproduce the theorem with cross-cuts.

(b.)

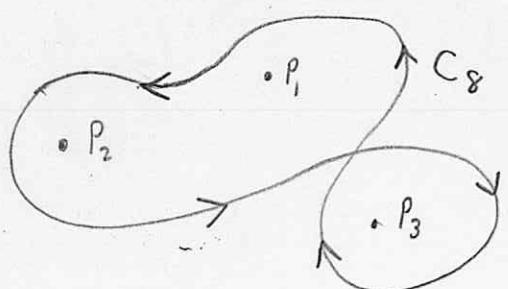
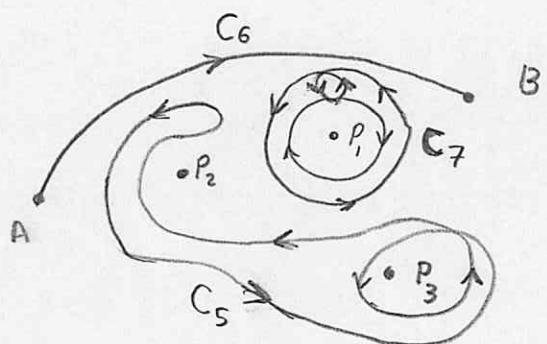
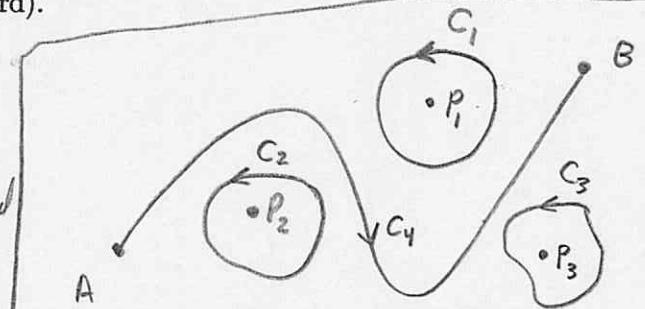


Observe that $C_6 \cup C_4$ enclose P_1 in CW sense

Thus, by Greene's Thm with holes,
 $\int_{C_6} \vec{F} \cdot d\vec{r} + \int_{-C_4} \vec{F} \cdot d\vec{r} = 1 \Rightarrow \int_{C_6} \vec{F} \cdot d\vec{r} = 4 - 1 = \boxed{3}$

(c.) $\int_{C_7} \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{-C_1} \vec{F} \cdot d\vec{r} = 1 - 1 = \boxed{0}$

(d.) $\int_{C_8} \vec{F} \cdot d\vec{r} = 1 + 2 - 3 = 0$ (used Greene's Thm with holes)
 (P_1, P_2) CCW (P_3) CW



Problem 195 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\mathbf{F}(x, y) = (x^2 + y^2, xy)$

(b.) $\mathbf{F}(x, y) = (2x + 3x^2y^2 + 5, 2x^3y)$

(c.) $\mathbf{F}(x, y) = (e^x, ye^{-y^2})$

(a.) $\partial_y(x^2 + y^2) = 2y$ and $\partial_x(xy) = y \therefore \partial_x Q \neq \partial_y P$
 Thus $\nexists \varphi$ s.t. $\vec{F} = \nabla \varphi$, \vec{F} not conservative.

(b.) $\partial_y(2x + 3x^2y^2 + 5) = 6x^2y$ and $\partial_x(2x^3y) = 6x^2y$.

We wish to find $\varphi = \varphi(x, y)$ for which,

$$\frac{\partial \varphi}{\partial x} = 2x + 3x^2y^2 + 5 \Rightarrow \varphi = x^2 + 5x + x^3y^2 + C_1(y)$$

$$\frac{\partial \varphi}{\partial y} = 2x^3y + \frac{\partial C_1}{\partial y} = 2x^3y \Rightarrow \frac{\partial C_1}{\partial y} = 0$$

Thus $\boxed{\varphi(x, y) = x^2 + 5x + x^3y^2}$

(c.) $\vec{F}(x, y) = \langle e^x, ye^{-y^2} \rangle$

$\therefore \boxed{\varphi(x, y) = e^x - \frac{1}{2}e^{-y^2}}$

Problem 196 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\mathbf{F}(x, y) = (e^{xy}, x^4y^3 + y)$

(b.) $\mathbf{F}(x, y) = \left(e^x + \frac{y}{1+x^2}, \arctan(x) + (1+y)e^y \right)$

(c.) $\mathbf{F}(x, y, z) = (yz + y + 1, xz + x + z, xy + y + 1)$

(a.) $\frac{\partial}{\partial y} (e^{xy}) = e^{xy} \frac{\partial}{\partial y} (xy) = xe^{xy} \neq \frac{\partial}{\partial x} (x^4y^3 + y).$

Therefore, \vec{F} not conservative.

(b.) Observe, $\frac{\partial \varphi}{\partial x} = e^x + \frac{y}{1+x^2} \Rightarrow \varphi = e^x + y \tan^{-1}(x) + C_1(y)$

$$\frac{\partial \varphi}{\partial y} = \tan^{-1}(x) + (1+y)e^y \Rightarrow \varphi = y \tan^{-1}(x) + ye^y + C_2(x)$$

Thus, $\boxed{\varphi(x, y) = e^x + y \tan^{-1}(x) + ye^y}$

this serves as potential fct, you can check $\nabla \varphi = \vec{F}$.

(c.) $\vec{F} = \langle yz + y + 1, xz + x + z, xy + y + 1 \rangle$ we can check

that $\nabla \times \vec{F} = \langle 0, 0, 0 \rangle \Rightarrow \exists \varphi$ on the simply connected \mathbb{R}^3 .

Observe $\underline{\varphi(x, y, z) = xyz + xy + yz + x + z}$

has $\nabla \varphi = \vec{F}$. Of course, we could derive
this by successive integrations in x, y and z .

Problem 197 Let $\vec{E}(x, y, z) = \frac{1}{(x^2+y^2+z^2)(3/2)}(x, y, z)$. Calculate $\nabla \cdot \vec{E}$ and $\nabla \times \vec{E}$. (use cartesians or spherical coordinates, your choice (the formulas for divergence and curl in sphericals are contained within the pdf on "VectorFieldDifferentiation" in course content). Plot this vector field (don't have to turn in plot, I trust you to do it), are your calculations surprising?

$$\vec{E} = \frac{1}{\rho^2} \hat{\rho} \quad \Rightarrow \quad \vec{E} = E_\rho \hat{\rho} + E_\phi \hat{\phi} + E_\theta \hat{\theta}$$

has $E_\rho = \frac{1}{\rho^2}, E_\phi = 0, E_\theta = 0.$

$$\begin{aligned}\nabla \cdot \vec{E} &= \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left[\rho^2 \frac{1}{\rho^2} \right] && \text{by formula on 377} \\ &= \frac{1}{\rho^2} \frac{\partial}{\partial \rho} (1) && \text{of my handwritten notes.} \\ &= \boxed{0} \quad \text{for } \rho \neq 0.\end{aligned}$$

$$\nabla \times \vec{E} = \det \begin{bmatrix} \hat{\rho} & \hat{\phi} & \hat{\theta} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ E_\rho & \underbrace{\rho E_\phi}_{0} & \underbrace{\rho \sin \phi E_\theta}_{0} \end{bmatrix} = \vec{0}$$

only $\frac{\partial}{\partial \phi}$ & $\frac{\partial}{\partial \theta}$

hit E_ρ hence zero
always results.

(Solved in separate pdf)

Problem 198 Let C be the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ (oriented counter-clockwise). Compute the line integral: $\int_C y^2 dx + x^2 dy$ two ways. First, compute the integral directly by parameterizing each side of the square. Then, compute the answer again using Green's Theorem.

Problem 199 Compute $\int_C \mathbf{F} \cdot d\mathbf{X}$ where $\mathbf{F}(x,y) = (y - \ln(x^2 + y^2), 2 \arctan(y/x))$ and C is the circle $(x-2)^2 + (y-3)^2 = 1$ oriented counter-clockwise.

Problem 200 prove the other half of Green's theorem. (I only proved half in lecture).

Problem 201 Calculate the integral below. Define C to be the CCW oriented triangle with vertices $(0,0)$, $(0,1)$ and $(1,1)$.

$$\oint_C (xy dx + x^2 dy)$$

Problem 202 Let C be the CCW oriented circle $x^2 + y^2 = 1$. Calculate

$$\oint_C (3y dx + 5x dy)$$

Problem 203 Let C be the CCW oriented rectangle with vertices $(0,0)$, $(a,0)$, (a,b) and $(0,b)$. Calculate

$$\oint_C x^2 dy$$

Problem 204 Suppose that f, g are continuously differentiable on an simply connected open set R .

Show that if C is any piecewise-smooth simple closed curve in R then

$$\oint [f(\vec{r})(\nabla g)(\vec{r}) + g(\vec{r})(\nabla f)(\vec{r})] \cdot d\vec{r} = 0$$