These problems are worth 1pt a piece at least. Feel free to use Mathematica or some other CAS to illustrate as needed.

Problem 101 Suppose S is the surface defined by $F(x, y, z) = xyz = 1$. Find the equation of the tangent plane and the parametrization of the normal line through $(1, 1, 1)$.

$$\nabla F = \langle yz, xz, xy \rangle$$

$$\nabla F (1,1,1) = \langle 1, 1, 1 \rangle$$

$$\frac{1(x-1) + 1(y-1) + 1(z-1) = 0}{OR \quad \boxed{x + y + z = 3}} \quad eq^n \text{ of tangent plane.}$$

$$\boxed{\vec{r}(t) = \langle 1, 1, 1 \rangle + t \langle 1, 1, 1 \rangle}$$

line through $(1,1,1)$ with direction $\langle 1, 1, 1 \rangle$ is the normal line.

Problem 102 Find a parametrization $\vec{X}$ of S from the previous problem which provides a patch in the locality of $(1, 1, 1)$. Use α, β for your parameters and find the normal-vector field $\vec{N}(\alpha, \beta)$ by computing $\vec{N}(\alpha, \beta) = \frac{\partial \vec{X}}{\partial \alpha} \times \frac{\partial \vec{X}}{\partial \beta}$. Do you obtain the same normal vector at $(1, 1, 1)$ with this patch?

$$\boxed{\vec{X}(\alpha, \beta) = \langle \alpha, \beta, \frac{1}{\alpha \beta} \rangle} \qquad \frac{xyz = 1}{\hookrightarrow z = \frac{1}{xy}}$$

most sets around $(1,1)$ which avoids $x = 0$ and $y = 0$ is a reasonable domain for $\vec{X}$

$$\vec{N}(\alpha, \beta) = \frac{\partial \vec{X}}{\partial \alpha} \times \frac{\partial \vec{X}}{\partial \beta} = \langle 1, 0, \frac{-1}{\alpha^2 \beta} \rangle \times \langle 0, 1, \frac{-1}{\alpha \beta^2} \rangle$$

$$\boxed{\vec{N}(\alpha, \beta) = \langle \frac{1}{\alpha^2 \beta}, \frac{1}{\alpha \beta^2}, 1 \rangle}$$

$$\vec{N}(1,1) = \langle 1, 1, 1 \rangle \qquad \text{yep, it's actually the same for my choice of } \vec{X}. \text{ However, it is possible you got } k \langle 1, 1, 1 \rangle = \vec{N}(\alpha, \beta) \text{ for some } k \neq 0$$

Problem 103 Label the solution set of $x^2 = y - z^2$ as M.

(a.) present M as a level-surface for some function F. Explicitly state the formula for F. Find the normal vector field on M.

(b.) parametrize M and once more find the normal vector field. This time find $\vec{N}$ explicitly in terms of your chosen parameters.

(a.) Let $F(x,y,z) = x^2 - y + z^2$ then $F^{-1}\{0\} = M$.

$$\boxed{\nabla F(x,y,z) = \langle 2x, -1, 2z \rangle} \leftarrow \text{normal vector field on } M.$$

(b.) Note $y = x^2 + z^2$ gives M use $y = R^2$ and
$x = R\cos\beta$, $z = R\sin\beta$ hence,

$$\vec{X}(R,\beta) = \langle R\cos\beta, R^2, R\sin\beta \rangle$$

$$\vec{N}(R,\beta) = \frac{\partial\vec{X}}{\partial R} \times \frac{\partial\vec{X}}{\partial\beta} = \langle \cos\beta, 2R, \sin\beta \rangle \times \langle -R\sin\beta, 0, R\cos\beta \rangle$$

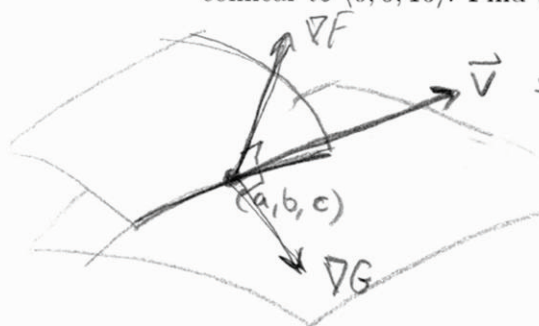$$= \langle 2R^2\cos\beta, -R\sin^2\beta - R\cos^2\beta, 2R^2\sin\beta \rangle$$

$$= \boxed{R \langle 2R\cos\beta, -1, 2R\sin\beta \rangle}$$

$$= R \langle 2x, -1, 2z \rangle$$

$$= R \nabla F \quad (\text{scalar multiple here})$$

Problem 104 Suppose a level surface $G(x,y,z) = 2$ has $\nabla G(x,y,z) = \langle x,y,z \rangle$ and another level surface $F(x,y,z) = 42$ with $\nabla F(x,y,z) = \langle 1,x,3 \rangle$. Suppose these surfaces intersect along some curve and at the point (a,b,c) the curve of intersection has tangent line with direction vector colinear to $\langle 0,0,10 \rangle$. Find (a,b,c).

ok, technically impossible ☺

$\vec{V}$ should $\perp$ to both ∇F and ∇G since the curve is on $G = 2$ and $F = 42$.

💡: $\vec{V} = \nabla F \times \nabla G$ at (a,b,c).

$$\nabla F \times \nabla G = \langle 1, x, 3 \rangle \times \langle x, y, z \rangle$$

$$= \langle xz - 3y, 3x - z, y - x^2 \rangle$$

Seek to solve $\langle 0,0,10 \rangle = \langle ac - 3b, 3a - c, b - a^2 \rangle$

Note, $3a = c \Rightarrow 0 = a(3a) - 3b \Rightarrow b = a^2 \Rightarrow \underline{b - a^2 = 0}$.

Oops. Well, sorry. (if you tried this I gave full credit.)

Problem 105 Consider the line-segment $\mathcal{L} = \overline{PQ}$ where $P = (1,2,3)$ to $Q = (5,0,-1)$.

    (a.) describe $\mathcal{L}$ parametrically as a path from $t = 0$ at P to $t = 1$ at Q.

    (b.) describe $\mathcal{L}$ parametrically as a path from $s = 0$ at Q to $s = 6$ at P.

    (c.) describe $\mathcal{L}$ as a graph. In particular, find $h(x)$ and $g(x)$ such that $f(x) = \langle g(x), h(x) \rangle$ and $graph(f) = \{ (x, f(x)) \mid x \in dom(f) \}$.

    (d.) describe $\mathcal{L}$ as a level-curve. In particular, find F such that $\mathbb{R}^3 \xrightarrow{F} \mathbb{R}^2$ and $\mathcal{L} = F^{-1}\{(0,0)\}$.

Remark: personally, I view the last two parts of the previous problem as less natural than the parametric presentation. It is in fact possible to present lines, surfaces, volumes etc... as either graphs, level-sets or as parametrized objects. Which to use depends on the context.

(a.) $\vec{r}(t) = P + t(Q-P)$ has $\vec{r}(0) = P$ and $\vec{r}(1) = Q$
applying this general trick to the given points,

$$\vec{r}(t) = (1,2,3) + t\left((5,0,-1) - (1,2,3) \right) \qquad \boxed{\mathcal{L} = \vec{r}([0,1])}$$

$$\boxed{\vec{r}(t) = \langle 1+4t, \ 2-2t, \ 3-4t \rangle, \quad 0 \le t \le 1}$$

(b.) rescale $\quad 0 \le t \le 1 \longrightarrow 0 \le 6t \le 6 \implies s = 6t$
so if we substitute $t = s/6$ we obtain the
desired parametrization. $\qquad\qquad \boxed{\mathcal{L} = \vec{\gamma}([0,6])}$

$$\boxed{\vec{\gamma}(s) = \langle 1 + \tfrac{4s}{6}, \ 2 - \tfrac{2s}{6}, \ 3 - \tfrac{4s}{6} \rangle, \quad 0 \le s \le 6}$$

Check: $\vec{\gamma}(0) = \langle 1,2,3 \rangle$ and $\vec{\gamma}(6) = \langle 1+4, 0, 3-4 \rangle$.

(c.) $\left. \begin{array}{l} x = 1+4t \\ y = 2-2t \\ z = 3-4t \end{array} \right\} \longrightarrow t = \dfrac{x-1}{4} = \dfrac{2-y}{2} = \dfrac{3-z}{4}$

                         Solve for y and z in terms of x,

                         $x-1 = 4-2y \implies 2y = 5-x \implies y = \dfrac{5-x}{2}$

                         $x-1 = 3-z \implies \underline{z = 4-x}$

$\boxed{\text{Thus, } f(x) = \langle \tfrac{1}{2}(5-x), \ 4-x \rangle \text{ will do the job.}}$
$\boxed{\text{We can see} \quad \mathcal{L} = graph(f) = \{ (x, f(x)) \mid x \in \mathbb{R} \}}$

(d.) $F(x,y,z) = (2y + x - 5, \ x + z - 4) = (0,0)$
$\qquad\qquad \implies \underline{\mathcal{L} = F^{-1}\{(0,0)\}}$ $\qquad\qquad\qquad \uparrow$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ symmetric eq's
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ~~for~~ line repackaged.

Problem 106 Ohms' Law says that $V = IR$ where V is the voltage of a battery which delivers a current I to a resistor R. As the current flows the battery will wear down and the voltage will drop. On the other hand, as the resistor heats-up the resistance will increase. Given that $R = 600$ ohms and $I = 0.04$ amp, if the resistance is increasing at a rate of 0.5 ohm/sec and the voltage is dropping at 0.01 volt/sec then what is the rate of change in the current I at this time.

$$dV = R\,dI + I\,dR$$

$$\Rightarrow \underbrace{\Delta V \cong R\Delta I + I\Delta R}_{\text{Solve for } \Delta I} \quad , \quad R = 600\,\Omega, \quad I = 0.04\,A$$
$$\Delta R = 0.5\,\Omega/s, \quad \Delta V = -0.01\,V/s$$

$$\Delta I = \frac{\Delta V - I\Delta R}{R} = \frac{-0.01\,V/s - (0.04A)(0.5\,\Omega/s)}{600\,\Omega}$$

$$\boxed{\Delta I = 5\times 10^{-5}\,A = 50\,\mu A}$$

Problem 107 Suppose that the temperature T in the xy-plane changes according to

$$\frac{\partial T}{\partial x} = 8x - 4y \quad \& \quad \frac{\partial T}{\partial y} = 8y - 4x.$$

where

Find the maximum and minimum temperatures of T ~~on~~ are the unit circle $x^2 + y^2 = 1$. To achieve this goal you should parametrize the the circle by $x = \cos t$ and $y = \sin t$ and calculate dT/dt and d^2T/dt^2 by the chain-rule. (you have no other option since the formula for T is not given!)

$$\frac{dT}{dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} = (8x - 4y)\frac{dx}{dt} + (8y - 4x)\frac{dy}{dt}$$

$$\frac{d^2T}{dt^2} = T_{xx}\left(\frac{dx}{dt}\right)^2 + T_x\frac{d^2x}{dt^2} + T_{yy}\left(\frac{dy}{dt}\right)^2 + T_y\frac{d^2y}{dt^2} \longrightarrow$$

Find critical points

$$T'(t) = (8\cos t - 4\sin t)(-\sin t) + (8\sin t - 4\cos t)\cos t$$
$$T'(t) = 4(\sin^2 t - \cos^2 t) = 4(-\cos(2t)) = 0 \qquad \begin{array}{l} 2t = \pi/2 \\ 2t = 3\pi/2 \\ 2t = 5\pi/2 \\ 2t = 7\pi/2 \end{array}$$

Thus, $t = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4,\ \dots$ critical pts.

$$T''(t) = 8\sin^2 t + (8\cos t - 4\sin t)(-\cos t) + 8(\cos^2 t) + (8\sin t - 4\cos t)(-\sin t)$$
$$T''(t) = 8 - 8(\cos^2 t + \sin^2 t) + 4\sin t \cos t + 4\cos t \sin t$$
$$\underline{T''(t) = 4\sin(2t)}$$

$$T''(\pi/4) > 0 \qquad \boxed{(1/\sqrt{2},\ 1/\sqrt{2})\ \text{at min.}}$$
$$T''(3\pi/4) < 0 \qquad \boxed{(-1/\sqrt{2},\ 1/\sqrt{2})\ \text{at max.}}$$

Then $T''(5\pi/4) > 0$ hence $\boxed{(-1/\sqrt{2},\ -1/\sqrt{2})\ \text{at min}}$ and $T''(7\pi/4) < 0$ $\boxed{(1/\sqrt{2},\ -1/\sqrt{2})\ \text{at max}}$

Problem 108 Suppose $f(u, v, w)$ is the formula for a differentiable f and $u = x - y$, $v = y - z$ and $w = z - x$. Show that $f_x + f_y + f_z = 0$.

$$f_x = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial x} = f_u + 0 - f_w$$

$$f_y = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial y} = -f_u + f_v + 0$$

$$f_z = \frac{\partial f}{\partial u}\frac{\partial u}{\partial z} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial z} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial z} = 0 - f_v + f_w$$

$$\therefore \quad f_x + f_y + f_z = f_u - f_w - f_u + f_v - f_v + f_w = 0.$$

$$\frac{\partial x}{\partial v} = \frac{\partial f}{\partial v}$$

Problem 109 Suppose $w = xy^2 + z^3$ and $x = f(u, v)$, $y = g(u, v)$ and $z = h(u, v)$ where f, g, h are differentiable functions. If $f(1, 2) = 2$ and $g(1, 2) = 3$ and $h(1, 2) = 4$ and $g_v(1, 2) = 0$, $h_v(1, 2) = 7$ and $f_v(1, 2) = 42$ calculate $\frac{\partial w}{\partial v}(1, 2)$.

$$\frac{\partial w}{\partial v} = \frac{\partial}{\partial v}\left(xy^2 + z^3\right) = y^2\frac{\partial x}{\partial v} + 2xy\frac{\partial y}{\partial v} + 3z^2\frac{\partial z}{\partial v}$$

$$\left(\frac{\partial w}{\partial v}\right)(1, 2) = \left(g(1,2)\right)^2 f_v(1,2) + 2 f(1,2) g(1,2) g_v(1,2) + 3\, h(1,2)^2\, h_v(1,2)$$

$$= 9(42) + 2(2)(3)\, 0 + 3(16)7$$

$$= \boxed{714}$$

Problem 110 Suppose $f(x, y) = x^2 - 3xy + 5$. A theorem states that for twice continuously differentiable f the error $E(x, y) = f(x, y) - L(x, y)$ in the linearization for each (x, y) in some rectangle R centered at (x_o, y_o) is bounded by

$$|E(x, y)| \leq \frac{1}{2} M \left(|x - x_o| + |y - y_o| \right)^2$$

where M bounds $|f_{xx}|$, $|f_{yy}|$ and $|f_{xy}|$ on R. In other words, if you can find such an M to bound the second order partial derivatives then the error is given by the inequality above.

(a.) find the linearization of f at $(2, 1)$.

(b.) bound the error $E(x, y)$ for the square $[1.9, 2.1] \times [0.9, 1.1]$

Remark: not that I plan to derive it this semester, but this is a consequence of the error estimate for the single-variable Taylor series as it applies to the construction of the multivariate Taylor expansion. The multivariate Taylor expansion derives from the chain-rule and Taylor's theorem from calculus II.

(a.) $\nabla f = \langle 2x - 3y, -3x \rangle \Rightarrow \nabla f(2,1) = \langle 1, -6 \rangle$

$\qquad\qquad\qquad\qquad\qquad\qquad f(2,1) = 4 - 3(2) + 5 = 3.$

$\qquad L_f(x, y) = 3 + 1(x-2) - 6y$

(b.) $f_{xx} = 2, \quad f_{yy} = 0, \quad f_{xy} = -3$

$\qquad$ clearly $|f_{xx}(x, y)|, |f_{yy}(x, y)|, |f_{xy}(x, y)| \leq 3$

Also since $x_o = 2$ and $y_o = 1$ if $(x, y) \in [1.9, 2.1] \times [0.9, 1.1]$

then $\quad 1.9 \leq x \leq 2.1$ and $0.9 \leq y \leq 1.1$

thus $\quad -0.1 \leq x - x_o \leq 0.1$ and $-0.1 \leq y - y_o \leq 0.1$

$\Rightarrow \quad |x - x_o| \leq 0.1$ and $|y - y_o| \leq 0.1$

Consequently, for $(x, y) \in [1.9, 2.1] \times [0.9, 1.1]$

$\qquad |E(x, y)| \leq \frac{3}{2}(0.1 + 0.1)^2 = \boxed{0.06}$

Problem 111 The area of a triangle is given by $A = \frac{1}{2}ab\sin\gamma$ where a, b are the lengths of two sides which have angle γ between them. Suppose that $\gamma = \pi/3 \pm 0.01$ and $a = 150 \pm 1$ ft and $b = 200 \pm 1$ ft. Find the corresponding uncertainty in the area. (leave answer as $A \pm \delta$ where the δ is calculated from the differential of A)

$$dA = \frac{1}{2}b\sin\gamma \, da + \frac{1}{2}a\sin\gamma \, db + \frac{1}{2}ab\cos\gamma \, d\gamma$$

To a good approximation,

$$\Delta A = \frac{1}{2}b\sin\gamma \, \Delta A + \frac{1}{2}a\sin\gamma \, \Delta b + \frac{1}{2}ab\cos\gamma \, \Delta\gamma$$

$$= \frac{1}{2}(200)\sin\left(\frac{\pi}{3}\right)(1) + \frac{1}{2}(150)\sin\left(\frac{\pi}{3}\right)(1) + \frac{1}{2}(150)(200)\cos\left(\frac{\pi}{3}\right)(0.01)$$

$$= 226.6 \ (ft)^2$$

$$\therefore \quad \boxed{A = 12,990 \pm 227 \quad ft^2}$$

Problem 112 Suppose you know $x = 3 \pm 0.01$ and $y = 4 \pm 0.01$ what are the corresponding polar coordinate ranges.

$$x = r\cos\theta \qquad\qquad r^2 = x^2 + y^2$$
$$y = r\sin\theta \qquad\qquad \theta = \tan^{-1}(y/x) + k \quad \leftarrow \begin{array}{l}\text{could be} \\ 0 \text{ or } \pi.\end{array}$$

$$2r\,dr = 2x\,dx + 2y\,dy$$

$$d\theta = \frac{1}{1+(y/x)^2}\left(\frac{-y}{x^2}\right)dx + \frac{1}{1+(y/x)^2}\left(\frac{1}{x}\right)dy = \frac{-y}{r^2}dx + \frac{x}{r^2}dy$$

Thus,

$$\Delta r = \frac{x}{r}\Delta x + \frac{y}{r}\Delta y = \frac{3}{5}(0.01) + \frac{4}{5}(0.01) = \boxed{0.014 = \Delta r}$$

$$\Delta\theta = \frac{-y}{x^2}\Delta x + \frac{x}{r^2}\Delta y = \frac{-4}{25}(0.01) + \frac{3}{25}(0.01) = \boxed{0.004 = \Delta\theta}$$

$$\boxed{\begin{array}{l} r = 5 \pm 0.014 \\ \Delta\theta = 0.927 \pm 0.004 \end{array}}$$

Problem 113 The kinetic energy in 2D-problem with cartesian coordinates is given by $K = \frac{1}{2}mv^2$ or explicitly in terms of the x, y velocities $\dot{x}, \dot{y}$ we have $K = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$. Calculate the formula for K in terms of polar coordinates r, θ and their velocities $\dot{r}, \dot{\theta}$.

$$\dot{x} = \frac{d}{dt}(r\cos\theta) = \dot{r}\cos\theta - r\sin\theta\,\dot{\theta}$$

$$\dot{y} = \frac{d}{dt}(r\sin\theta) = \dot{r}\sin\theta + r\cos\theta\,\dot{\theta}$$

$$\dot{x}^2 = \dot{r}^2\cos^2\theta - 2r\sin\theta\cos\theta\,\dot{r}\dot{\theta} + r^2\sin^2\theta\,\dot{\theta}$$

$$\dot{y}^2 = \dot{r}^2\sin^2\theta + 2r\sin\theta\cos\theta\,\dot{r}\dot{\theta} + r^2\cos^2\theta\,\dot{\theta}$$

$$\Rightarrow K = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \boxed{\tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)}$$

$$y = w - \sin t - x^2 + z$$

Problem 114 Suppose $w = x^2 + y - z + \sin(t)$ and $x + y = t$. Calculate the following constrained partial derivatives:

(a.) $\left(\dfrac{\partial w}{\partial y}\right)_{x,z}$ (b.) $\left(\dfrac{\partial w}{\partial y}\right)_{z,t}$ (c.) $\left(\dfrac{\partial w}{\partial z}\right)_{x,y}$ (d.) $\left(\dfrac{\partial w}{\partial z}\right)_{y,t}$

(a.) $\left(\dfrac{\partial w}{\partial y}\right)_{x,z} = 2x\underbrace{\left(\dfrac{\partial x}{\partial y}\right)_{x,z}}_{0} + \underbrace{\left(\dfrac{\partial y}{\partial y}\right)_{x,z}}_{1} - \underbrace{\left(\dfrac{\partial z}{\partial y}\right)_{x,z}}_{0} + \cos t\underbrace{\left(\dfrac{\partial t}{\partial y}\right)_{x,z}}_{1} = \boxed{1 + \cos t}$

since $t = x + y$

(b.) $\left(\dfrac{\partial w}{\partial y}\right)_{z,t} = 2x\left(\dfrac{\partial x}{\partial y}\right)_{z,t} + \underbrace{\left(\dfrac{\partial y}{\partial y}\right)_{z,t}}_{1} - \underbrace{\left(\dfrac{\partial z}{\partial y}\right)_{z,t}}_{0} + \cos t\underbrace{\left(\dfrac{\partial t}{\partial y}\right)_{z,t}}_{0}$

$= 1 + \underbrace{2x\left(\dfrac{\partial x}{\partial y}\right)_{z,t}}$

since z, y independent and t, y independent.

to calculate this directly I need to solve $x = x(y, z, t)$
Note that $x = t - y$
Hence $\left(\dfrac{\partial x}{\partial y}\right)_{z,t} = -1$.   (the other eq^n would be a poor choice here !)

$$\therefore \boxed{\left(\dfrac{\partial w}{\partial y}\right)_{z,t} = 1 - 2x}$$

Problem 114 , method of differentials

$w = x^2 + y - z + \sin t$

$t = x + y$

$$\boxed{\begin{aligned} dw &= 2x\,dx + dy - dz + \cos t\,dt \\ dt &= dx + dy \end{aligned}}\; - (*)$$

For parts (a) we have x, y, z independent, w, t dep.
thus solve for dw & dt. (in terms of dx, dy, dz)

$$dw = 2x\,dx + dy - dz + \cos t\,[dx + dy]$$

$$\underline{dw = (2x + \cos t)dx + (1 + \cos t)dy - dz} \quad \& \quad \underline{dt = dx + dy}$$

$$\left(\frac{\partial w}{\partial x}\right)_{y,z} = 2x + \cos t$$

$$\boxed{\left(\frac{\partial w}{\partial y}\right)_{x,z} = 1 + \cos t}$$ (as we found using
direct computation on last page)

$$\left(\frac{\partial w}{\partial z}\right)_{x,y} = -1$$

$$\left(\boxed{\left(\frac{\partial t}{\partial x}\right)_{y,z} = 1 \;,\; \left(\frac{\partial t}{\partial y}\right)_{x,z} = 1 \;,\; \left(\frac{\partial t}{\partial z}\right)_{x,y} = 0}\right) \leftarrow$$

Part (b.) says z, t, y indep. and x, w dep.
need to solve (*) for dx & dw in terms of dy, dz, dt

$$dx = dt - dy$$

$$dw = 2x\,dx + dy - dz + \cos t\,dt = 2x(dt - dy) + dy - dz + \cos t\,dt$$

$$dw = (1 - 2x)dy - dz + (2x + \cos t)dt$$

$$\boxed{\left(\frac{\partial w}{\partial y}\right)_{z,t} = 1 - 2x} \qquad \text{(again same as before)}$$

PROBLEM 114 continued (differential method)

(c.) to calculate $\left(\frac{\partial w}{\partial z}\right)_{x,y}$ I need $w = w(z,x,y)$ & $t = t(z,x,y)$

we need to solve (*) for dw and dt

$$dt = dx + dy \quad \text{(done)}$$

$$dw = 2x\,dx + dy - dz + \cos t\,\boxed{dt}$$

$$\Rightarrow \quad dw = 2x\,dx + dy - dz + \cos t \,(dx + dy)$$

$$\Rightarrow \quad dw = (2x + \cos t)\,dx + (\cos t + 1)\,dy - dz$$

can read off that,

$$\left(\frac{\partial w}{\partial x}\right)_{y,z} = 2x + \cos t$$

$$\left(\frac{\partial w}{\partial y}\right)_{z,x} = \cos t + 1$$

$$\boxed{\left(\frac{\partial w}{\partial z}\right)_{x,y} = -1}$$

(d.) $\boxed{\left(\frac{\partial w}{\partial z}\right)_{y,t} = -1}$ $\left(\text{from } dw = (1-2x)\,dy - dz + (2x + \cos t)\,dt\right)$