Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 81 Your signature below indicates you have:

(a.) I have read §4.8 and Chapter 5 of Cook: ____________________.

(b.) I have attempted homeworks from Salas and Hille as listed below:________________.

The following homeworks from the text are good rudimentary skill problems. These are not collected or graded. However, they are all usually odd problems thus there are answers given within Salas, Hille and Etgen's text:

§ 15.5 #'s 5, 9, 15, 19, 23, 25, 29, 31, 33, 37, 41, 47, 49

§ 15.6 #'s 1, 3, 9, 13, 19, 25, 36

Problem 82 Let $f(x,y) = \frac{x+y}{\sqrt{x^2+y^2}}$. Find ∇f in terms of polar coordinates (use the polar frame $\hat{r}, \hat{\theta}$ as discussed in my notes).

Problem 83 Let $f(x,y,z) = \frac{x+y+z}{x^2+y^2+z^2}$. Find ∇f in terms of spherical coordinates (your answer should use the spherical frame $\rho, \widehat{\phi}, \widehat{\theta}$ as discussed in my notes).

Problem 84 Suppose $\vec{F} = r^2\widehat{r} + \frac{1}{r}\theta\widehat{\theta} + ze^{z^2}\,\widehat{z}$. Find f such that $\nabla f = \vec{F}$.

Problem 85 Find all critical points of $f(x,y) = 2x - x^2 - y^2$

Problem 86 Find all critical points[1] of $f(x,y) = 3x^2 + xy - y^2 + 5x - 5y + 4$.

Problem 87 Find all critical points of $f(x,y) = xy^{-1} - yx^{-1}$. The answer here is involves an infinite set of points so it would be proper to use set-notation to express the answer.

Problem 88 Find all critical points of $f(x,y,z) = \cos(x^2 + y^2 + z^2)$. The answer here is involves an infinite set of points so it would be proper to use set-notation to express the answer.

Problem 89 Consider $f(x,y) = x^2 + kxy + 4y^2$ where k is a constant. For what value(s) of k is the second derivative test inconclusive?

Problem 90 Consider $f(x,y) = x^2 - y^2$. Find the absolute extrema for f on the rectangle $[-1,1] \times [0,2]$.

Problem 91 Let a, b be constants. Find the absolute maximum value of $f(x,y) = \frac{(ax+by+x)^2}{x^2+y^2+1}$.

Problem 92 Use the method of Lagrange multipliers to find the point on the plane $x + 2y - 3z = 10$ which is closest to the point $(10, 9, 8)$.

[1] Salas and Hille call these "stationary points" when ∇f exists

Problem 93 Apply the method of Lagrange multipliers to solve the following problem: Find the distance from $(6,0)$ to the parabola $x^2 = 4y$.

Problem 94 Find the maximum and minimum values for $f(x,y) = x^2 - y^2 - 1$ on the region bounded by the triangle with vertices $(1,1), (3,5)$ and $(6,0)$.

Problem 95 Find the multivariate power series expansion for $f(x,y) = (x+y)e^x \sin(y)$ centered at $(0,0)$ to second order.

Problem 96 Given that $f(x,y) = 3 + x^2 - y^2 + 13x^3 + 42y^3 + \cdots$ decide if $(0,0)$ is a critical point of f and if $(0,0)$ is critical then classify the nature of the critical point as maximum, minimum or saddle.

Problem 97 Given that $f(x,y) = 3 + x + (x-1)^2 + (y-3)^2 + \cdots$ decide if $(1,3)$ is a critical point of f and if $(1,3)$ is critical then classify the nature of the critical point as maximum, minimum or saddle.

Problem 98 Find the multivariate power series expansion of $f(x,y) = \sin(x+y)$ to second order about the point (π, π).

Problem 99 Find the multivariate power series explansion of $f(x,y,z) = \frac{1}{1+x+y+z}$ centered about $(0,0,0)$ to second order.

Problem 100 Suppose objective function $f(x,y)$ has an extremum on $g(x,y) = 0$. Show that F defined by $F(x,y,\lambda) = f(x,y) - \lambda g(x,y)$ recovers the extremum as a critical point. From this viewpoint, the adjoining of the multiplier converts the constrained problem in n-dimensions to an unconstrained problem in $(n+1)$-dimensions (you can easily generalize your argument to $n > 2$).

Bonus: An armored government agent decides to investigate a disproportionate use of electricity in a gated estate. Foolishly entering without a warrant he find himself at the mercy of Ron Swanson (at $(1,0,0)$), Dwight Schrute (at $(-1,1,0)$) and Kakashi (in a tree at $(1,1,3)$). Supposing Ron Swanson inflicts damage at a rate of 5 units inversely proportional from the square of his distance to the agent, and Dwight inflicts constant damage at a rate of 3 in a sphere of radius 2. If Kakashi inflicts a damage at a rate of 5 units directly proportional to the square of his distance from his location (because if you flee it only gets worse the further you run as he attacks you retreating) then where should you assume a defensive position as you call for back-up? What location minimizes your damage rate? Assume the ground is level and you have no jet-pack and/or antigravity devices.