Math 231 Mission 6

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

## Problem 76 Your PRINTED NAME below indicates you have:

- (a.) I have read Chapter 6 of Cook: \_\_\_\_\_\_.
- (b.) I have attempted homeworks from Salas and Hille as listed below:

Same deal as Mission 1. Enjoy:

**Problem 77** Calculate 
$$\int_0^1 \int_x^{x^3} y^2 \, dy \, dx$$

**Problem 78** Calculate 
$$\int_{0}^{1} \int_{0}^{x} \int_{x^{2}+y^{2}}^{x^{3}+y^{3}} 2z \, dz \, dy \, dx$$

**Problem 79** Let  $f(x,y) = x^2 + y^3$ . Find the average of f on the square  $[0,1] \times [0,1]$ .

Problem 80 Reverse the order of integration in order to calculate the following integral:

$$\int_0^1 \int_0^1 \frac{2}{1+x^4} \, dx \, dy.$$

- **Problem 81** Let B be the solid region bounded by x = 0, y = 0, z = 0 and the plane 2x + 2y 3z = 1. Calculate the volume of B.
- **Problem 82** Let R be the regions in quadrant I bounded by  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$  and y = x and y = 3x. Calculate  $\iint_R \sqrt{x^2 + y^2} \, dA$  by changing the integration to polar coordinates.

- **Problem 83** Find the volume bounded by the cylinder  $x^2 + y^2 = 1$  and z = 2 + x + y and z = 1.
- **Problem 84** Let B be a ball of radius R centered at the origin. Calculate  $\iiint_R e^{-\rho^3} dV$ .
- **Problem 85** Find the centroid of the region bounded by  $r = 4\cos\theta$  and r = 2.
- **Problem 86** Suppose the density of a solid sphere B of radius R is given by  $\delta = \alpha \rho^c$  where  $\alpha, c$  are constants and  $\rho = \sqrt{x^2 + y^2 + z^2}$ . The units of the constants are chosen as to give  $\delta = \frac{dm}{dV}$  units of mass per volume. The moment of intertia with respect to the z axis is given by  $I = \iiint_B r^2 \delta dV$  where  $r^2 = x^2 + y^2$ . Calculate I as a function of the constants c and  $\alpha$ .
- **Problem 87** Calculate  $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left(12-x^2-y^2-\sqrt{x^2+y^2}\right) dy dx$  by changing to an integral in polar coordinates.
- **Problem 88** Let P be the parallelogram with vertices (0,0),(0,1),(2,4),(2,5). Calculate

$$\iint_{P} \sqrt{2x(y-2x)} \, dA$$

by making a substitution of x = 2u and y - 2x = v. Notice, this substitution changes P to  $[0,1] \times [0,1]$  in u,v space. Thanks to Brigg's and Cochran section 14.7 for this example.

**Problem 89** Use the Jacobian to relate the volume element  $du d\phi d\theta$  to the Cartesian volume element dx dy dz given that

$$x = u \cos \theta \cosh \phi,$$
  $y = u \sin \theta \cosh \phi,$   $z = u \sinh \phi.$ 

**Problem 90** Calculate the hypervolume of the set  $\{(x, y, z, t) \mid x^2 + y^2 + z^2 + t^2 \le 1\}$