Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 121 Your signature below indicates you have:

(a.) I have read §7.1 – 7.5 of Cook: ____________________.

(b.) I have attempted homeworks from Salas and Hille as listed below:________________.

The following homeworks from the text are good rudimentary skill problems. These are not collected or graded. However, they are all usually odd problems thus there are answers given within Salas, Hille and Etgen's text:

§ 17.1 #'s 3, 7, 11, 15, 21, 27

§ 17.2 #'s 3, 5, 9, 17, 25

§ 17.3 #'s 1, 9

§ 17.4 #'s 9, 11, 21, 29, 33

§ 17.5 #'s 1, 5, 7, 11, 15, 17, 21, 31

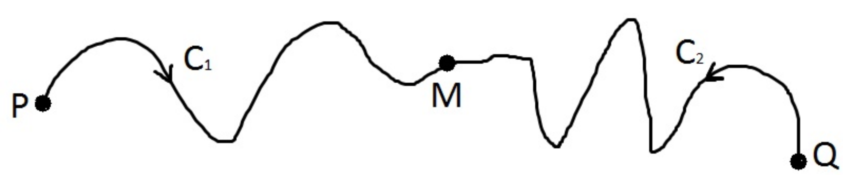
§ 17.8 #'s 1, 5, 9, 13, 14.

Problem 122 Let $\vec{F}(x, y, z) = \langle e^x - y, x + \cos(y), 3\rangle$ calculate $\nabla \bullet \vec{F}$ and $\nabla \times \vec{F}$.

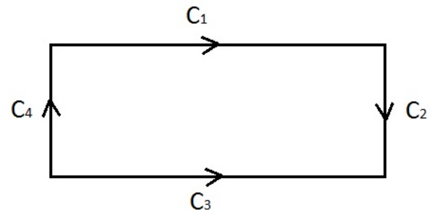
Problem 123 Suppose the charge density along a wire is given by $\lambda(x, y, z) = bx^2 + cy^3$ for constants b and c. Furthermore, the wire follows the path C with parametric equations $x = R\cos t$, $y = R\sin t$ and $z = mt$ for constants m, R and $0 \leq t \leq 2\pi$. Find the net charge on C by calculating the integral with respect to arclength $\int_C \lambda ds$.

Problem 124 Let C be the non-linear helix with parametric equations $x = 2\cos t$, $y = 2\sin t$ $z = t^2$ for $0 \leq t \leq 2\pi$. Calculate $\displaystyle\int_C \vec{F} \bullet d\vec{r}$ where $\vec{F}(x, y, z) = \langle x,\ y,\ x^2 + y^2 + z\rangle$

Problem 125 Suppose $\displaystyle\int_{C_1} \vec{F} \bullet d\vec{r} = 1$ and $\displaystyle\int_{C_2} \vec{F} \bullet d\vec{r} = 2$. Let C be the path from P to M to Q which follows the same set of points as C_1 and C_2. Calculate $\displaystyle\int_C \vec{F} \bullet d\vec{r}$.



Problem 126 Calculate $\int_C \vec{F} \bullet d\vec{r}$ where $\vec{F}(x, y) = \langle y,\, x + y \rangle$ and $C = C_1 \cup C_2 \cup C_3 \cup C_4$ is pictured below and has corners at $(0, 0), (4, 0), (4, 1)$ and $(0, 1)$:



Accomplish this task by calculating the line integrals of $\vec{F}$ on C_1, C_2, C_3, C_4 separately.

Problem 127 Let $\vec{F} = (\phi + \rho^2)\widehat{\phi}$ where I use the spherical coordinate system. Let C be the curve of intersection of $\theta = \pi/4$ and $\rho = 2$ oriented to travel in the direction of decreasing z. Calculate $\int_C \vec{F} \bullet d\vec{r}$.

Problem 128 Let C be a path from $(0, 1)$ to $(2, 3)$. Calculate $\int_C (1 + x)dx + (2 + y)dy$.

Problem 129 Find the work done by the force $\vec{F}(x, y, z) = \langle y + x^2, x, z^3 \rangle$ on a mass as it moves from $(0, 0, 0)$ to $(1, 2, 3)$.

Problem 130 Find k such that $\int_{\partial R} ydx + kxdy$ gives the area of the simple, positively oriented region R with boundary ∂R.

Problem 131 Let D be the unit disk $D = \{(x, y) \mid x^2 + y^2 \leq \pi\}$. Calculate

$$\iint_D \left(\frac{\partial}{\partial x}\left[x \sin(x^2 + y^2)\right] - \frac{\partial}{\partial y}\left[y^2 \sin(x^2 + y^2)\right] \right) dA.$$

Problem 132 Let C be the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$ (oriented counter-clockwise). Compute the line integral: $\int_C y^2\, dx + x^2\, dy$ two ways. First, compute the integral directly by parameterizing each side of the square. Then, compute the answer again using Green's Theorem.

Problem 133 Let C be the boundary of the region R bounded above by $y = x$ and below by $y = x^2 - 2x$. Calculate $\oint_C 3xy\, dx + 2x^2\, dy$ by application of Green's Theorem.

Problem 134 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\vec{F}(x, y) = \langle 2x - \sin(x + y^2), -2y \sin(x + y^2) \rangle$

(b.) $\vec{F}(x, y) = \langle -y, x \rangle$

(c.) $\vec{F}(y, z) = \langle z + y^2, y + z^3 \rangle$

Problem 135 Let $\vec{F} = \langle 0, 0, -mg \rangle$ where m, g are positive constants and suppose $\vec{F}_f = -b\vec{T}$ where v is your speed and b is a constant and $\vec{T}$ is the unit-vector which points along the tangential direction of the path. This is a simple model of the force of kinetic friction, it just acts opposite your motion. Find the work done by $\vec{F}_f + \vec{F}$ as you travel up the helix $\vec{r}(t) = \langle R\cos(t), R\sin(t), t \rangle$ for $0 \leq t \leq 4\pi$.

Problem 136 A vector field $\vec{F}$ gives a specific vector $\vec{F}(x, y, z)$ for each point (x, y, z) in the domain of $\vec{F}$. An **integral curve** or **flow line** of $\vec{F}$ is a path $t \to \vec{r}(t)$ for which $\vec{F}(\vec{r}(t)) = \frac{d\vec{r}}{dt}$ along each t in the domain of the path. In particular, if $\vec{F} = \langle P, Q, R \rangle$ and $\vec{r} = \langle x, y, z \rangle$ then the integral curve must solve:

$$P = \frac{dx}{dt}, \quad Q = \frac{dy}{dt}, \quad R = \frac{dz}{dt}$$

where P, Q, R are evaluated at $(x(t), y(t), z(t))$. Find the integral curves of the vector fields:

(a.) $\vec{G}(x, y, z) = \langle a, b, c \rangle$ where a, b, c are constants,

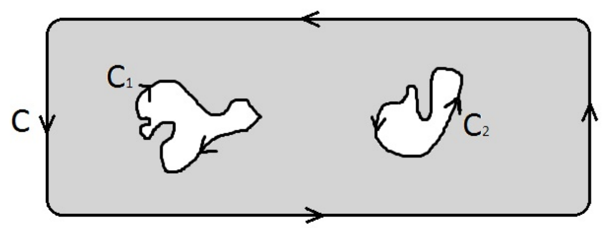
(b.) $\vec{F}(x, y, z) = \langle -y, \, x, \, 1 \rangle$.

To solve the differential equations which arise in part (b.) here, you want to eliminate all but one dependent variable (these are x, y, z here) and solve the resulting ordinary differential equation. See §18.3 of Salas and Hille for the formulas, or ask me once you get the differential equations set-up.

Problem 137 Prove Proposition 7.2.4 part (*iii.*)

Problem 138 Green's Theorem: complete the proof by showing the details for part **II.** (see page 362).

Problem 139 Suppose we are given $\int_{C_1} \vec{F} \bullet d\vec{r} = 1$ and $\int_{C_2} \vec{F} \bullet d\vec{r} = -3$. Calculate $\int_C \vec{F} \bullet d\vec{r}$ given that $\vec{F}$ is conservative (locally) on the domain between C and the curves C_1, C_2. Is $\vec{F}$ conservative on $\mathbb{R}^2$? Explain.



Problem 140 Suppose $\vec{F}(x, y, z) = \langle y, x^2, 3 \rangle$ and suppose S is the surface parametrized by $\vec{r}(u, v) = \langle u, \, 3 + v \,, 2 - u^2 \rangle$ for $(u, v) \in [0, 1] \times [0, 1]$. Calculate the flux of $\vec{F}$ through S. That is, calculate $\iint_S \vec{F} \bullet d\vec{S}$.