

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 141 Your signature below indicates you have:

(a.) I have read §7.6 – 7.10 of Cook: _____.

(b.) I have attempted homeworks from Salas and Hille as listed below:_____.

The following homeworks from the text are good rudimentary skill problems. These are not collected or graded. However, they are all usually odd problems thus there are answers given within Salas, Hille and Etgen's text:

§ 17.6 #'s 5, 7, 9, 11, 13, 19, 27

§ 17.7 #'s 1, 3, 9, 15, 21, 23, 31, 35

§ 17.8 #'s 21, 25

§ 17.9 #'s 1, 5, 9, 13, 17

§ 17.10 #'s 1, 3, 5, 8, 11, 15

Problem 142 Find the surface area of torus with radii $A, R > 0$ and $R \geq A$ parametrized by

$$\vec{X}(\alpha, \beta) = \left\langle [R + A \cos(\alpha)] \cos(\beta), [R + A \cos(\alpha)] \sin(\beta), A \sin(\alpha) \right\rangle$$

for $0 \leq \alpha \leq 2\pi$ and $0 \leq \beta \leq 2\pi$.

Problem 143 Let S be the outward oriented unit-sphere. Calculate $\iint_S \langle x^3, y^3, z^3 \rangle \cdot d\vec{S}$.

Problem 144 Consider a thin-shell of constant density δ . Let the shell be cut from the cone $x^2 + y^2 - z^2 = 0$ by the planes $z = 1$ and $z = 2$. Find (a.) the center of mass and (b.) the moment of inertia with respect to the z -axis.

Problem 145 Find the flux of $\vec{F}(x, y, z) = \langle z^2, x, -3z \rangle$ through the parabolic cylinder $z = 4 - y^2$ bounded by the planes $x = 0$, $x = 1$ and $z = 0$. Assume the orientation of the surface is outward, away from the x -axis.

Problem 146 Find the flux of $\vec{F}(x, y, z) = \langle -x, -y, z^2 \rangle$ through the conical frustrum $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$ with outward orientation.

Problem 147 Suppose \vec{C} is a constant vector. Let $\vec{F}(x, y, z) = \vec{C}$ find the flux of \vec{F} through a surface S on plane with nonzero vectors \vec{A}, \vec{B} . In particular, the surface S is parametrized by $\vec{r}(u, v) = \vec{r}_o + u\vec{A} + v\vec{B}$ for $(u, v) \in \Omega$.

Problem 148 Let $\phi = \pi/4$ define a closed surface S with $0 \leq \rho \leq 2$. Find the flux of

$$\vec{F}(\rho, \phi, \theta) = \phi^2 \hat{\rho} + \rho \hat{\phi} + \hat{\theta}$$

through the outward oriented S .

Problem 149 Consider the closed cylinder $x^2 + y^2 = R^2$ for $0 \leq z \leq L$. Find the flux of

$$\vec{F}(r, \theta, z) = \theta \hat{z} + z \hat{\theta} + r^2 \hat{r}$$

out of the cylinder.

Problem 150 Let $\vec{F}(x, y, z) = \langle y^2 + z^2, x^2 + z^2, x^2 + y^2 \rangle$. Find the work done by \vec{F} around the CCW (as viewed from above) triangle formed from the intersection of the plane $x + y + z = 1$ and the coordinate planes. (use Stoke's Theorem)

Problem 151 Let $\vec{F} = \langle 2x, 2y, 2z \rangle$ and suppose S is a simply connected surface with boundary ∂S a simple closed curve. Give two arguments (one by Stokes', the other by Gauss' theorem) that $\int_{\partial S} \vec{F} \cdot d\vec{r} = 0$.

Problem 152 Suppose S is the union of the cylinder $x^2 + y^2 = 1$ for $0 \leq z \leq 1$ and the disk $x^2 + y^2 \leq 1$ at $z = 1$. Suppose \vec{F} is a vector field such that

$$\nabla \times \vec{F} = \left\langle \sinh(z)(x^2 + y^2), ze^{xy + \cos(x+y)}, (xz + y) \tan^{-1}(z) \right\rangle.$$

Calculate the flux of $\nabla \times \vec{F}$ through S .

Problem 153 Let E be the cube $[-1, 1]^3$. Calculate the flux through ∂E of the vector field

$$\vec{F}(x, y, z) = \langle y - x, z - y, y - x \rangle$$

(please use the divergence theorem!)

Problem 154 Suppose E is the spherical shell $R_1 \leq \rho \leq R_2$ and suppose $\vec{F}(x, y, z) = \nabla \times \vec{A}$ for some everywhere smooth vector field \vec{A} . Show that the flux through $\rho = R_1$ is the same as the flux through $\rho = R_2$ by applying the divergence theorem to the spherical shell.

Problem 155 Show that for a simple solid E with consistently oriented boundary ∂E if f, g are twice differentiable on some open set containing E then

$$(a.) \iiint_E (f \nabla^2 g + \nabla f \cdot \nabla g) dV = \iint_{\partial E} f \nabla g \cdot d\vec{S}.$$

$$(b.) \iiint_E (f \nabla^2 g - g \nabla^2 f) dV = \iint_{\partial E} [f \nabla g - g \nabla f] \cdot d\vec{S}.$$

Problem 156 Suppose $\nabla^2 f = 0$ on a simply connected solid E . If $f|_{\partial E} = 0$ then what can you say about f throughout E ?

(here $|_{\partial E}$ denotes restriction of f to the subset ∂E . In particular this means you are given that $f(x, y, z) = 0$ for all $(x, y, z) \in \partial E$.)

Problem 157 Suppose $b : \mathbb{R}^3 \rightarrow \mathbb{R}$ is a particular function and $\nabla^2 f = b$ on a simply connected solid region E . If g is another such solution ($\nabla^2 g = b$) on E then show that $f = g$ on E .
The equation $\nabla^2 f = b$ is called Poisson's Equation. When $b = 0$ then it's called Laplace's Equation. You are showing the solution to Poisson's Equation is unique on a simply connected solid region. Hint: use the last problem's result on $f - g$.

Problem 158 We gave definitions for curl and divergence which were based in cartesian coordinates. Some authors actually use the identities below to define curl and divergence. Naturally, if you use these as definitions then the question of what div and curl mean are easily answered. However, on the other hand, in that approach you have no simple formula to calculate curl or div until you have mastered both surface and line integrals. I wanted to talk about curl and div before that point so for that reason I did not take these as definitions.

- (a.) Assume E is a volume with piecewise smooth, outward oriented, boundary ∂E where E contains the point P . Then if we shrink the volume down to P we obtain the divergence of a differentiable \vec{F} as follows:

$$\operatorname{div}(\vec{F})(P) = \lim_{V \rightarrow 0^+} \frac{1}{V} \iint_{\partial E} \vec{F} \cdot d\vec{S}.$$

Show the formula above is true by an argument involving the divergence theorem.

- (b.) Assume S is a surface with piecewise smooth, consistently oriented, boundary ∂S where E contains the point P . Then if we shrink the surface to P we obtain the curl of the the vector field in the direction of the normal \hat{n} to S at P as follows:

$$\left[\operatorname{curl}(\vec{F})(P) \right] \cdot \hat{n} = \lim_{A \rightarrow 0^+} \frac{1}{A} \oint_{\partial S} \vec{F} \cdot d\vec{r}$$

Problem 159 Suppose we have a vector field expressed in cylindrical coordinates; $\vec{F} = F_r \hat{r} + F_\theta \hat{\theta} + F_z \hat{z}$. Calculate the formulas for

- (a.) $\operatorname{div}(\vec{F})$
 (b.) $\operatorname{curl}(\vec{F})$

By applying the boxed formulas of the previous problem to appropriate volumes and loops. For the divergence you want to think about a little part of a cylinder which corresponds to a small change in r, θ and z . For the curl you want to think about three loops. To pick out the z -component you should fix z and allow r and θ to sweep out a little sector. I'll draw the pictures for you if you ask.

Problem 160 Find a function Φ and a vector field \vec{A} such that $\vec{F} = \nabla\Phi + \nabla \times \vec{A}$ where $\vec{F}(x, y, z) = \langle e^x - y, x + \cos(y), 3 \rangle$.

Bonus: Suppose $\vec{J} = \sigma \vec{E}$ (this is Ohm's Law for current density, the constant σ is the **conductivity**). Show that Maxwell's equations yield the equation below:

$$\nabla^2 \vec{E} = \mu_o \sigma \frac{\partial \vec{E}}{\partial t} + \mu_o \epsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}$$

This is called the **telegrapher's equation**.