

Please work the problems in the white space provided and clearly box your solutions. You are allowed one $3'' \times 5''$ notecard. Enjoy! This quiz has 3pts of bonus credit.

Problem 1 Convert $f(x, y, z) = \frac{z}{x^2+y^2+z^2}$ to spherical coordinates and calculate ∇f in terms of the spherical coordinate frame. (leave answer in terms of $\hat{\rho}, \hat{\phi}, \hat{\theta}$ and the spherical coordinates ρ, ϕ, θ)

Problem 2 Find the multivariate power series expansion of $f(x, y) = e^{x+y^2}$ at $(0, 0)$ to second order. Is $(0, 0)$ a critical point? If so, classify the nature of the point.

Problem 3 Find all critical points of $f(x, y, z) = \cos(x + y + z)$.

Problem 4 Find the extreme values of $f(x, y) = x^2 + y^2 + x^2e^y$ on the disk $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$.

Problem 5 Suppose $f(x, y) = 6 + 3(x - 1)^2 - 4(y + 2)^2 - 10(x - 1)(y + 2) + \dots$. Determine if $(1, -2)$ is a critical point and if it is use the second derivative test to classify the nature of the point as min/max/saddle if possible.

Problem 6 Calculate $\int_0^1 \int_0^{x^2} e^{x^3} dy dx$

Problem 7 Express the integral $\int_0^1 \int_{x^3}^{\sqrt{x}} f(x, y) dy dx$ as an iterated integral based on a Type II region.

Problem 8 Calculate $\iiint_C (x^2 + y^2)z dV$ where C is the solid cylinder of radius $r = 3$ for $0 \leq z \leq 1$.

Problem 9 Let $B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 4, z \geq 0\}$. Calculate $\iiint_B (x^2 + y^2) dV$.

Problem 10 Suppose the mass per unit area is given by $\sigma(x, y) = x$. Find the center of mass for the rectangular region $R = [0, 1] \times [0, 1]$.

Problem 11 Find the area of the elliptical region $x^2/a^2 + y^2/b^2 \leq 1$.