

Please work the problems in the white space provided and clearly box your solutions. You are allowed one $3'' \times 5''$ notecard. Enjoy! This quiz has 3pts of bonus credit.

Problem 1 (9pts) Let $f(x, y) = x + x^2y^2$. Find the following:

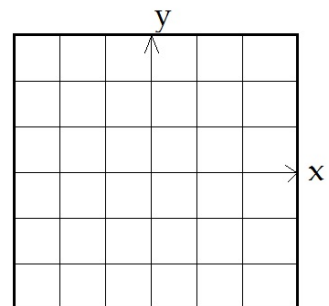
(a.) $\frac{\partial f}{\partial x}$

(b.) $\frac{\partial f}{\partial y}$

(c.) the equation of the tangent plane at $(1, 2)$.

Problem 2 (6pts) Let $f(x, y) = \frac{1}{1 + x^2 + y^3}$. Find the rate of change in f at $(1, 0)$ in the $\langle 8, -6 \rangle$ direction.

Problem 3 (6pts) Let $f(x, y) = x^2 + y^2$. Plot level curves $f(x, y) = k$ for $k = 1, 2, 9$ on the graph template given below. Furthermore, find (with calculus) the direction in which f is locally constant at $(1, 1)$ and indicate why it is reasonable in view of your plotted level curves.



Problem 4 (10pts) Let $z = \sin(xy^2)$. Furthermore, $x = \alpha + \beta^2$ and $y = \beta^2 \sin(\beta)$. Use the multivariate chain rule to calculate $\frac{\partial z}{\partial \alpha}$ and $\frac{\partial z}{\partial \beta}$.

Problem 5 (6pts) Suppose $h_x(1, 2) = 3$ and $h_y(1, 2) = 7$. In addition, $x(u, v) = uv^2$ and $y(u, v) = u^2 - v$. Let $g(u, v) = h(x(u, v), y(u, v))$ and calculate $\frac{\partial g}{\partial u}(1, -1)$.

Problem 6 (9 pts) Let $\vec{r}(s, t) = \langle t \cos 3s, t \sin 3s, 4s \rangle$ be the parametrization of a surface M . Find the normal vector field to M .

Problem 7 (6pts) A surface $Ax^2 + By^3 + Cz^4 = 5$ has a normal line at $(1, -1, 1)$ with direction vector $\langle 12, 12, 12 \rangle$. Find the values of the constants A, B and C .

Problem 8 (6 pts) Calculate the limit below (if it doesn't exist then explain why)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{y - x^2}{x^2 + y^2}$$

Problem 9 (6pts) You are stuck on the surface $xy^2z^3 = 1$. At the point $(1, 1, 1)$ you measure your x -velocity to be $2ft/s$ and your y -velocity is $-3ft/s$. Assuming units of ft and s find $\frac{dz}{dt}$.

Problem 10 (6pts) Imagine yourself a pig. Ron Swanson is holding a barbeque of epic size in a heavily misty park. At the same time a bagpipe club is playing at an embarrassing volume. You can't see or hear in effect. The smell of meat is measured to roughly follow $M(x, y) = x^2 - 2xy$ at $(1, 1)$. In which direction should you travel to **get away** from the feast?

Problem 11 (6pts) Suppose $w = x^2 + yz^3$ where $\cos(xyz) = y^2 + z^2$. Calculate $\left(\frac{\partial w}{\partial x}\right)_y$.

Problem 12 (6pts) Calculate dF for $F(x, y, z) = \ln \left[e^x z^{z^2} [\cosh^2(xyz) - \sinh^2(xyz)] \sqrt{\sec(y^2)} \right]$. Simplify your answer.

Problem 13 (10 pts) Solve $(2xy^2 + e^x)dx + (2x^2y + y \cos(y^2))dy = 0$.

Problem 14 (3pts) Let $\omega = -ydx + xdy$. Is this form closed? Is this form exact?

Problem 15 (5pts) Suppose f, g are smooth real-valued functions of n -variables. Show that

$$\nabla(fg) = (\nabla f)g + f(\nabla g).$$

Problem 16 (4pts) Let $\vec{A}(x, y, z) = \left\langle 2x, -2y, \frac{1}{z^2 + 1} \right\rangle$. If possible, find Φ such that $\nabla\Phi = \vec{A}$.

Problem 17 (4pts) Let $\vec{B}(x, y, z) = \langle 2x + z, x^3, y \rangle$. If possible, find Ψ such that $\nabla\Psi = \vec{B}$.