Same instructions as Mission 1. Thanks!

Problem 21 Your signature below indicates you have:

(a.) I read pages 43-65 of Stillwell's *Elements of Number Theory*: ____________.

(b.) I read Cook's handout on *Modular Arithmetic*: ____________.

Problem 22 Let $a, b, c \in \mathbb{Z}$. **Prove:** if $a|b$ and $b|c$ then $a|c$.

Problem 23 Let $a, b, c \in \mathbb{Z}$. **Prove:** if $a|b$ and $c|d$ then $ab|cd$.

Problem 24 Convert $(89156)_{10}$ to base 8 notation.

Problem 25 exercise 3.1.4 on page 45

Problem 26 exercise 3.2.1 on page 48

Problem 27 exercise 3.3.4 on page 51

Problem 28 exercise 3.4.3 on page 53

Problem 29 Define $f(x) = 3x + 2$ where $x \in \mathbb{Z}_4$ and $f(x) \in \mathbb{Z}_5$. Is f so defined a function ?

Problem 30 Write the addition and multiplication tables for $\mathbb{Z}_4 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$.

Problem 31 Explain why $H = \{\overline{1}, \overline{2}, \overline{3}\}$ does **not** form a group with respect to multiplication mod 4.

Problem 32 For which $a \in \mathbb{Z}$ is it the case that $\overline{4a + 3} = \overline{6}$ in $\mathbb{Z}_{12}$?

Problem 33 Find the remainder of 5^{10} divided by 19.

Problem 34 Find the final hexidecimal digit of $1! + 2! + 3! + 4! + 5! + 6! + \cdots + 1000!$.
Useful reminder: Recall a hexidecimal number is a base 16 representation $n = a_o + a_1(16) + a_2(16)^2 + \cdots + a_k(16)^k$ where the hexidecimal digits $a_0, a_1, \dots, a_k \in \{0, 1, \dots, 15\}$. However, we use notation $10 = A, 11 = B, 12 = C, 13 = D, 14 = E$ and $15 = F$ to write such numbers. For example, $AF = 10(16) + 15 = 175$. To be more pedantic, $(AF)_{16} = (175)_{10}$.

Problem 35 How many zeros are there at the end of 200! in decimal notation?

Problem 36 Show that the greatest common divisor of two even integers is even.

Problem 37 Prove or disprove: for any positive integer n the fraction $\frac{15n+4}{10n+3}$ is in *lowest terms*. For example, $\frac{3}{6}$ is not in lowest terms as $\frac{3}{6} = \frac{1}{2}$. A fraction in lowest terms cannot be further reduced as a single fraction. (of course, with egyptian fractions and continued fractions we have many other options, but that is beside the point here)

Problem 38 **Prove by induction on k:** If p is prime and $p|a_1a_2\ldots a_k$ then $p|a_i$ for at least one a_i.

Problem 39 Show that $\sqrt[3]{5}$ is irrational. Hint $1^3 = 1$ and $2^3 = 8$.

Problem 40 Show that if a positive integer m is not a perfect square, then $\sqrt{m}$ is irrational.