Same instructions as Mission 1. Thanks!

Problem 41 Your signature below indicates you have:

(a.) I read pages 66-75 of Stillwell's *Elements of Number Theory*: ____________.

(b.) I read Cook's handout on *Basic Algebra in* $\mathbb{Z}_n$: ____________.

Problem 42 exercise 3.3.3 page 51

Problem 43 exercise 3.5.1 and 3.5.2 page 55

Problem 44 An ancient method to find primes is known as the sieve Eratosthenes. See this wikipedia article for example. Verify the list of primes shown on the wikipedia article and find the smallest integer n such that $[1000000, n]$ has 7 primes via experimenting with the prime-finding command in Sage.

Problem 45 By this time we ought to have shown that any prime divisor of n is at most $\sqrt{n}$. Use this insight and other math we have discussed to prove or disprove that 203 is prime.

Problem 46 exercise 3.6.4 page 57

Problem 47 exercise 3.7.1 and 3.72 page 59.

Problem 48 exercise 3.7.3 page 59

Problem 49 exercise 3.8.3 page 61

Problem 50 find the least positive residue of 3^{100000} modulo 35. Use Euler's Theorem.

Problem 51 exercise 4.1.2 page 70

Problem 52 exercise 4.3.1 page 72

Problem 53 exercise 4.4.1 page 73

Problem 54 I chose $m, b \in \mathbb{Z}_{26}$ to make the affine shift cipher $f(x) = mx + b$ based on the alphabet code $A = 0, B = 1, \cdots Z = 25$. You are given that the message "CAT" is encrypted to "KEJ". Find m and b and decode the message "REDMJU".

Problem 55 Prove that $a(a + 1)(2a + 1)$ is divisible by 6 for every integer a.

Problem 56 Show that $f(x) = x^5 - x^2 + x - 3$ has no integer roots by showing $f(x) \equiv 0$ $(mod\ 4)$. It is convenient for this sort of problem to use the least absolute residues: $\mathbb{Z}_4 = \{\overline{-1}, \overline{0}, \overline{1}, \overline{2}\}$ as they permit faster calculation of $f(x)$.

Problem 57 Decide if $3x \equiv 5$ $(mod\ 7)$ or $12x \equiv 15$ $(mod\ 22)$ has solutions. Find the solutions for the congruence which has solutions.

Problem 58 Simultaneously solve $x \equiv 2$ $(mod\ 3)$, $x \equiv 3$ $(mod\ 5)$, $x \equiv 2$ $(mod\ 7)$. Notice, you can use the Sage to check your answer here. See page 30 of this free legal pdf by Stein no joke

Problem 59 Find the solutions of $f(x) = x^3 + 4x^2 + 19x + 1 \equiv 0 \ (mod \ 5^2)$

Problem 60 Find all the roots of $x^{18} + 4x^{14} + 3x + 10 \equiv 0 \ (mod \ 21)$