Math 321	Quiz 1 (takehome, due start of class Monday)

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 1 If possible, calculate $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \\ 7 & 8 \end{bmatrix}$

Problem 2 If possible, calculate $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^2$

Problem 3 If possible, calculate $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}^{-1}$

Problem 4 If possible, calculate $\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}^8$

Problem 5 If possible, calculate $e_1 \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Problem 6 If possible, calculate $\begin{bmatrix} a & b \\ c & d \end{bmatrix} e_1$

Problem 7 Let $Q(x, y) = x^2 + 2xy - y^2$. Find $A = A^T$ for which $Q(v) = v^T Av$ where $v^T = [x, y]$.

Problem 8 Let A, B, C be square matrices. Define $[A, B] = AB - BA$. Show

$$[A+B, C] = [A, C]+[B, C].$$

Problem 9 Let A be a nonzero square matrix such that $A^2 \neq 0$ yet $A^3 = 0$. Is A invertible? Is $I + A$ invertible?

Problem 10 Suppose A is a symmetric matrix. Prove A^n is symmetric for all $n \in \mathbb{N}$.

Problem 11 Suppose there exist matrices X which are multipliable with A, B such that $AX = BX$. Is it true, possible, or false that $A = B$?

Problem 12 Solve (freestyle, I judge your answer not your work here, but, keep in mind, this might be a test question so wolfram alpha etc. is really not the style which helps you prepare)

$$x - y + z + t = 14, \qquad x - z = 1, \qquad z + t = 12, \qquad x - t = 3.$$

Problem 13 Suppose that the matrix below is the augmented coefficient matrix for a linear system of equations $Av = b$ where $v = (x_1, x_2, x_3, x_4)$,

$$[A|b] = \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & 9 \\ 1 & 2 & 3 & 0 & 14 \\ 1 & -1 & 0 & 0 & -1 \end{array}\right] \qquad \text{and you're given:} \qquad rref[A|b] = \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 4 \\ 0 & 1 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Perform the following two tasks:

(i.) write the **system of equations** $Av = b$ in "scalar" form.

(ii.) give the **general solution** with pivot variables as the dependent variables.

(iii.) give the **solution set** parametrized by the non-pivot variables.

Problem 14 Find the solution set of the system of equations below using row-reduction.

$$x + y + z = 2, \qquad 2x + 2y + 2z = 4, \quad y - z = 2$$

Use the non-pivot variables to parametrize the solution set.

Problem 15 Is $(1,2,0,0) \in \text{span}\{(1,0,2,2),(0,1,2,2),(1,1,0,0)\}$? Explain.

Problem 16 Let $S = \{(1,0,2,2),(0,1,2,2),(1,1,0,0)\}$. Is S a LI set?

Problem 17 Let $n \geq 5$. Suppose $\{v_1, v_2, v_3, v_4\} \subset \mathbb{R}^n$ such that $\text{rref}[v_1|v_2|v_3|v_4|w] = [e_1|2e_1|e_2|e_1+e_2|e_5]$. Use the given information to answer the following:

(a) is $\{v_1, v_2\}$ LI ?

(b) is $\{v_1, v_3\}$ LI ?

(c) is $\{v_2, v_4\}$ LI ?

(d) is $w \in \text{span}\{v_1, v_2, v_3, v_4\}$?

Problem 18 Let $k \in \mathbb{R}$ and define $S = \{(1,1,0),(0,1,1),(1,k,1)\}$. For which values of k is it true that $\mathbb{R}^3 = \text{span}(S)$?

Problem 19 Consider $S = \{(1,2,3),(1,1,0),(1,1,1),(3,4,4)\}$. Find all LI subsets $T \subseteq S$ such that $\text{span}(T) = \mathbb{R}^3$.

Problem 20 Find the set of all possibly degenerate cubic polynomials whose graphs contain the points $(1,1)$, $(2,1)$ and $(-1,0)$.

Problem 21 Suppose $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$. Furthermore, suppose

$$EA = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 6 & 7 & 8 \end{bmatrix}$$

Which elementary matrix E makes the equation above true?

Problem 22 Consider the the system of equations $Av = 0$ where

$$A = \begin{bmatrix} 1 & 4 & 6 \\ -1 & -3 & -6 \\ 2 & 8 & 13 \end{bmatrix}$$

and v is notation for $v = [x, y, z]^T$. Solve this system by explicit row reduction.

Problem 23 Is the matrix below invertible? If so, find the inverse.

$$A = \begin{bmatrix} 1 & 4 & 6 \\ -1 & -3 & -6 \\ 2 & 8 & 13 \end{bmatrix}$$

Problem 24 Consider $T(x, y, z) = (x + 4y + 6z,\ -x - 3y - 6z,\ 2x + 8y + 13z)$. Show that T is a linear bijection on $\mathbb{R}^3$. Notice, the previous problem saves you calculation here.

Problem 25 A kind robot tells you that $rref\left[\begin{array}{ccc|c} 1 & 0 & 1 & 16 \\ 2 & 2 & 0 & 20 \\ 3 & 1 & 0 & 24 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 9 \end{array}\right]$. Define vectors as follows:

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad w = \begin{bmatrix} 16 \\ 20 \\ 24 \end{bmatrix}$$

Answer the following questions: (you do not have to explain, just this once)

(a) Is $w \in span(\{v_1, v_2, v_3\})$? If it is then provide the linear combination of the vectors v_1, v_2, v_3 which yields w.

(b) Is $\{v_1, v_2, v_3\}$ a linearly independent set of vectors? (Yes No Maybe)

(c) Is $\{v_1, v_2, v_3, w\}$ a linearly independent set of vectors? (Yes No Maybe)

Problem 26 Define a function $T : \mathbb{R}^3 \to \mathbb{R}^3$ by the formula below:

$$T(x, y, z) = (x + z,\ 2x + 2y, 3x + y).$$

(a) find the standard matrix for T

(b) is T linear? Why?

(c) is T one-one ?

(d) is $w = (16, 20, 24) \in T(\mathbb{R}^3)$?

Problem 27 Prove that $(c_1 + c_2)v = c_1 v + c_2 v$ for all $v \in \mathbb{R}^n$ and $c_1, c_2 \in \mathbb{R}$.

Problem 28 Let $T(x, y, z) = (x + y - z, y + z)$ and $S(a, b) = (2a + b, a - b, 3a + b)$. If possible, calculate the standard matrix for $S \circ T$ and give the formula for $S \circ T$. If possible, calculate the standard matrix for $T \circ S$ and give the formula for $S \circ T$.

Problem 29 Find the set of all $n \times n$ matrices which commute with all $n \times n$ matrices. That is, find all matrices $X \in \mathbb{R}^{n\times n}$ such that $AX = XA$ for all $A \in \mathbb{R}^{n\times n}$.

Problem 30 Find all 2×2 matrices which commute with $B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$