MATH 321	QUIZ 3 (TAKEHOME, DUE START OF CLASS 4-27)

Copying answers and steps is strictly forbidden. Working together is encouraged, share ideas not calculations. Show work on other paper. Box your answers where appropriate. Please do not fold. Thanks!

Problem 1 Suppose $A \in \mathbb{R}^{3\times 3}$ and $B \in \mathbb{R}^{4\times 4}$ and $\det(A) = 2$ and $\det(B) = 13$. Calculate:

$$\det \left[\begin{array}{c|c} -A & 0 \\ \hline 0 & 3B \end{array}\right].$$

Problem 2 Define $W = \text{span}\{(1, 0, 1, 1),\ (0, 2, 2, 3)\}$. Find an orthonormal basis γ_1 for W and an orthonormal basis γ_2 for $W^\perp$. Set $\beta = \gamma_1 \cup \gamma_2$ and calculate $[v]_\beta$ for $v = (2, 2, 2, 2)$. Also, calculate $\text{Proj}_W(v)$ and $\text{Proj}_{W^\perp}(v)$

Problem 3 Let $T(f(x)) = f''(x)$ and define $V = \text{span}\{e^{2x},\ e^{3x},\ \sin(x)\}$. Show that T is diagonalizable by choosing an e-basis β for which $[T]_{\beta\beta}$ is diagonal. Calculate $\det(T)$ and $\text{trace}(T)$.

Problem 4 Find the real Jordan form of $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix}$.

Problem 5 Find the formula for $Q(v)$ in terms of eigencoordinates y_1, y_2, y_3 given that

$$Q(v) = x^2 + y^2 + z^2 + 4xy - 4xz + 4yz$$

for the usual Cartesian coordinates $v = (x, y, z)$.

Problem 6 Find the eigenvalues and a basis for each eigenspace of $A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

Problem 7 Find an orthonormal basis for $W^\perp$ where $W = \text{span}\{(1, 1, 1, 0, 0), (2, 2, 2, 2, 2), (0, 1, 1, , 0, 2)\}$

Problem 8 If $A = \text{diag}\left(\begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \right)$ where this notation indicates that A is block-diagonal with the diagonal blocks as given. Find the eigenvalues of A and state the algebraic and geometric multiplicity of each eigenvalue. In addition, find the characteristic and minimal polynomials for A. Exhibit the cycle Tableau for appropriate nilpotent maps associated to A.

Problem 9 Let $T : V \to V$ be a linear transformation and $\dim(V) = 6$ over $\mathbb{R}$. Find the characteristic and minimal polynomials of T given that: there exist linearly independent vectors v_1, v_2, v_3, v_4, v_5 in V such that:

$$T(v_1) = 3v_1, \quad T(v_2) - 3v_2 = v_1, \quad T(v_3 + iv_4) = (2 + i)(v_3 + iv_4), \quad T^2(v_5) = 0.$$

Problem 10 Suppose A and B are nilpotent matrices for which $AB = BA$. Is $cA + B$ nilpotent for any $c \in \mathbb{R}$? Prove or disprove.