Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Include some white space between problems and work the problems in the order they are assigned. Please do not fold. Thanks!

GRADING: 3pts per problem, 5pts just for turning it in on time and following formatting directions.

Problem 1 Saff and Snider §1.1#4.
Prove that if $z_1 z_2 = 0$ then $z_1 = 0$ or $z_2 = 0$.

Problem 2 Saff and Snider §1.1#20.
Solve each of the following equations for z.

(a.) $iz = 4 - zi$

(b.) $\frac{z}{1-z} = 1 - 5i$

(c.) $(2 - i)z + 8z^2 = 0$

(d.) $z^2 + 16 = 0$

Problem 3 Saff and Snider §1.1#27.
Prove the *binomial formula* for complex numbers $z, w \in \mathbb{C}$:

$$(z + w)^n = \sum_{k=0}^{n} \binom{n}{k} z^{n-k} w^k$$

where n is a positive integer and the *binomial coefficients* are given by $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$.

Problem 4 Saff and Snider §1.3#5a, b. Calculate

(a.) $\left|\dfrac{1 + 2i}{-2 - i}\right|$

(b.) $\left|\overline{(1 + i)}(2 - 3i)(4i - 3)\right|$

Problem 5 Taken from Saff and Snider §1.3#7. Find the argument of each complex number and write each in polar form:

(a.) $\dfrac{-1}{2}$

(b.) $-3 + 3i$

(c.) $(\sqrt{3} - i)^2$

(d.) $\dfrac{-\sqrt{7}(1 + i)}{\sqrt{3} + i}$

Problem 6 Saff and Snider §1.3#8.
Show $0 \neq z_1, z_2 \in \mathbb{C}$ satisfy $|z_1 + z_2| = |z_1| + |z_2|$ if and only if $arg(z_1) = arg(z_2)$.

Problem 7 Saff and Snider §1.4#4. (about finding polar form of given complex numbers)

Problem 8 Saff and Snider §1.4#19. (neat result about sum of complex numbers)

Problem 9 Saff and Snider §1.4#20. (identities about sum of cosines and sines)

Problem 10 Saff and Snider §1.5#8. (quadratic equation, do not just quote quadratic formula here, you need to work it out)

Problem 11 Saff and Snider §1.5#9. (cubic equation)

Problem 12 Saff and Snider §1.5#10. (quartic equation and factoring)

Problem 13 Consider the set of points in $\mathbb{C}$ defined by $|z - 1 - i| + |z + 1 + i| = 4$.

(a.) without any calculation, explain geometrically what this curve is, what do we call such curves ?

(b.) find the Cartesian equation for this curve without any radicals. (if the formula is not familiar, use some CAS to graph the curve)

Problem 14 Define $\overline{x + iy} = x - iy$. Prove $\overline{Z_1 Z_2} = \overline{Z_1}\ \overline{Z_2}$ for all $Z_1, Z_2 \in \mathbb{C}$.

Problem 15 Let us set aside our nice notation $x_1 + iy_1$ and return to the annoying notation (x_1, y_1) to prove a few elementary properties of complex multiplication. Proofs should be based on the usual definition of vector addition and real scalar multiplication and the following definition of complex multiplication:

$$(x_1, y_1) \star (x_2, y_2) = (x_1x_2 - y_1y_2, x_1y_2 + y_1x_2).$$

Using the parenthetical notation show the following claims:

(a.) $(1, 0) \star (x, y) = (x, y)$ for all (x, y)

(b.) $(x, y) \star (x, y) = (-1, 0)$ if and only if $(x, y) = (0, \pm 1)$

(c.) complex multiplication is distributive;
$(x_1, y_1) \star [(x_2, y_2) + (x_3, y_3)] = (x_1, y_1) \star (x_2, y_2) + (x_1, y_1) \star (x_3, y_3)$

(d.) let $c \in \mathbb{R}$, $[c(x_1, y_1)] \star (x_2, y_2) = (x_1, y_1) \star [c(x_2, y_2)]$

Let me decode what you've shown above. In the other notation, the notation we will almost always use from here on out, you will have shown:

(a.) $1z = z$ for all $z = x + iy$,

(b.) $z^2 = -1$ if and only if $z = \pm i$,

(c.) complex multiplication is distributive; $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$,

(d.) $cz_1z_2 = z_1cz_2$ for $c \in \mathbb{R}$ and $z_1, z_2 \in \mathbb{C}$.