Copying answers and steps is strictly forbidden. Same instructions as Mission 1. Do not fold. Thanks!

Problem 11 Your signature below indicates you have:

- **(a.)** I have read Chapter 2 of Gamelin: ____________________.
- **(b.)** I have read Cook's Guide to Chapter 2:________________.

Problem 12 Show $\sin(z+w) = \cos(z)\sin(w) + \sin(z)\cos(w)$ for all $z, w \in \mathbb{C}$. Then use the relation of $\sinh(z), \cosh(z)$ to $\sin(z), \cos(z)$ to derive a corresponding identity for $\sinh(z+w)$.

Problem 13 Solve $\sinh z = i$ and write the infinite solution set in terms of $n \in \mathbb{Z}$.

Problem 14 Find a branch $g(w)$ of $w^{1/3}$ which serves as the inverse function of $f(z) = z^3$ with $\text{dom}(f) = \{z \in \mathbb{C}^\times \mid \text{Arg}(z) \in [0, 2\pi/3)\}$. Relate $g(w)$ to the principal branch $\sqrt[3]{w}$. Recall, the principal branch is defined by $\sqrt[3]{w} = \sqrt[3]{|w|}\exp\left(\frac{i\text{Arg}(w)}{3}\right)$ for each $w \in \mathbb{C}^-$ and serves as the inverse function to $h(z) = z^3$ with $\text{dom}(h) = \{z \in \mathbb{C}^\times \mid \text{Arg}(z) \in (-\pi/3, \pi/3)\}$. Draw a few pictures to organize your thoughts.

Problem 15 Let $S_\alpha = \mathbb{C} - e^{i\alpha}[0, \infty)$; this is $\mathbb{C}$ with the ray at $\theta = \alpha$ removed. Furthermore, define $\text{Log}_\alpha(z) = \ln|z| + i\text{Arg}_\alpha(z)$ where we define $\text{Arg}_\alpha(z)$ to be the single element found in $arg(z) \cap (\alpha, \alpha + 2\pi]$. Calculate the discontinuity at $\theta = \alpha$ by following any circle $\gamma(t) = Re^{it}$ with parameter values $t \in (\alpha, \alpha + 2\pi]$. Compare $\text{Log}_\alpha(\gamma(\alpha + 2\pi))$ and $\lim_{t \to \alpha^+} \text{Log}_\alpha(\gamma(t))$.

Problem 16 Let $\mathbb{H} = \{z \in \mathbb{C} \mid \mathfrak{Im}(z) > 0\}$.

- **(a.)** show with a sketch $1 + i$ is an interior point of $\mathbb{H}$.
- **(b.)** show every point in $\mathbb{H}$ is an interior point of $\mathbb{H}$. Your argument may be guided by a sketch, but it should also be supported with explicit inequality arguments.

Problem 17 Let $a_n = z^n$ for $n \in \mathbb{N}$.

- **(a.)** for which $z \in \mathbb{C}$ is a_n a bounded ?
- **(b.)** for which $z \in \mathbb{C}$ does a_n form a convergent sequence ?

Problem 18 Consider the annulus A defined by $1 \leq |z - 2| \leq 2$. Carefully sketch A and, if possible, find sets $A_1, A_2 \subseteq A$ for which $A_1 \cup A_2 = A$ and A_1, A_2 is star-shaped. If it is not possible, then try to accomplish the same with A_1, A_2, A_3 defined similarly. What is the minimum number of $A_1, \dots, A_k$ needed for this construction? In your answer, do indicate the star-centers as to be clear on your claim.

Problem 19 Let $g : \text{dom}(g) \subseteq \mathbb{C} \to \mathbb{C}$. **Prove the following:**
If $\lim_{h \to 0} g(h)/|h| = 0$ then $\lim_{h \to 0} g(h)/h = 0$.

Problem 20 Let $f(z) = 1/z^2$.

- **(a.)** Show $f'(z) = -2/z^3$ by calculating the limit of the difference quotient.
- **(b.)** Find the derivative of $f(z) = 1/z^2$ for $z \neq 0$ via the Theorem of Caratheodory. See my notes for several similar problems including $f(z) = 1/z$ etc.