Same instruction as Mission 1. Enjoy !

Problem 16 Show $e^z = w$ if and only if $z \in \log(w)$. Use your result to solve $e^z = -7i$.

Problem 17 Let $z, w \in \mathbb{C}$. Prove $\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$ directly from the definitions $\sinh z = \frac{1}{2}(e^z - e^{-z})$ and $\cosh z = \frac{1}{2}(e^z + e^{-z})$. Recall we have proved $e^z e^w = e^{z+w}$ for all $z, w \in \mathbb{C}$.

Problem 18 Calculate $log(-2-3i)$ and find $Log(-2-3i)$.

Problem 19 Calculate the following values or sets of values:

(a.) $\cos(3+i)$

(b.) $(1+i)^{1/3}$

(c.) $(1+i)^{1+i}$

Problem 20 Find the solution set of the equations below:

(a.) $\sin(2iz+1) = 0$

(b.) $Log(z^2-1) = \frac{i\pi}{2}$

(c.) $e^{2z} + e^z + 1 = 0$

Problem 21 Are the solution sets of the following open, closed, neither ? You can argue by a picture and a sentence or two, I don't expect rigorous proofs here.

(a.) $Im(z) \leq -2$

(b.) $|z-i| < 2$

(c.) $1 < |z - e^{i\pi/3}| < 3$

Problem 22 Let $A = \{z \in \mathbb{C} \mid 1 \leq |z-1| \leq 3\}$.

(a.) show geometrically that A is connected,

(b.) explain geometrically why A is not star-centered

(c.) is A a compact set ?

Problem 23 Prove $S = \{z = x+iy \in \mathbb{C} \mid y > 1\}$ is an open set. Your proof should include arguments showing there exists an open disk around an arbitrary point in S.

Problem 24 Saff and Snider §2.1#1. (calculation of component functions)

Problem 25 Saff and Snider §2.1#3. (range of given complex functions)

Problem 26 Saff and Snider §2.2#12. (discontinuity of the Argument)

Problem 27 Let $F(x,y) = (x^2-y^2, -2xy)$ for all $(x,y) \in \mathbb{R}^2$.

(a.) Calculate J_F (the Jacobian matrix of F)

(b.) Use $x = \frac{1}{2}(z+\bar{z})$ and $y = \frac{1}{2i}(z-\bar{z})$ to express the formula for F in terms of $z = x+iy$ and $\bar{z} = x - iy$.

Problem 28 Let $F(x, y) = (2(x^2 - y^2) - 6xy, 4xy + 3(x^2 - y^2))$ for all $(x, y) \in \mathbb{R}^2$.

(a.) Calculate J_F (the Jacobian matrix of F)

(b.) Use $x = \frac{1}{2}(z+\bar{z})$ and $y = \frac{1}{2i}(z-\bar{z})$ to express the formula for F in terms of $z = x+iy$ and $\bar{z} = x - iy$. The answer here has the form cz^2 for an appropriate choice of c.

Problem 29 Let $f(z) = z^4$. Show that $f'(z) = 4z^3$ for all $z \in \mathbb{C}$ in four ways:

(a.) by direct calculation of the limit of difference quotient

(b.) by the method of Caratheodory

(c.) by applying the Cauchy Riemann equation theorem

(d.) by the Wirtinger calculus.

Problem 30 Let $f(z)$ be complex differentiable at z_o. Prove f is continuous at z_o.

Hint: if you remember the proof that differentiable implies continuous at a point in Calculus I then that proof can be easily adapted to this problem. But, the easier way by far is to use the Theorem of Caratheodory to characterize the existence of $f'(z_o)$.