Problem 21 Let $f(x,y) = x^y$ and calculate the differential df. Let $\gamma(t) = (t,t)$ and use the chain rule to calculate $(f \circ \gamma)'(t)$ (*note the chain rule is accomplished via multiplication of Jacobian matrices* $[df]$ *and* $[\gamma]$ *in this context*). Contrast this calculation to the calculation you used in calculus I to find $\frac{d}{dx}(x^x)$.

Problem 22 Suppose $X : \mathbb{R}^2 \to \mathbb{R}^3$ is given by $X(s,t) = (x(s,t), y(s,t), z(s,t))$ and $\bar{X} : \mathbb{R}^2 \to \mathbb{R}^3$ is given by $\bar{X}(\bar{s},\bar{t}) = (\bar{x}(\bar{s},\bar{t}), \bar{y}(\bar{s},\bar{t}), \bar{z}(\bar{s},\bar{t}))$. Suppose further that there exists some notation changing map $T : \mathbb{R}^2 \to \mathbb{R}^2$ where $X = \bar{X} \circ T$ meaning:

$$\bar{X}(T(s,t)) = X(s,t)$$

where the notation $T(s,t) = (\bar{s}(s,t), \bar{t}(s,t))$ yields

$$\bar{X}((\bar{s}(s,t), \bar{t}(s,t))) = X(s,t)$$

Find $\frac{\partial X}{\partial s}$ and $\frac{\partial X}{\partial t}$ in terms of $\frac{\partial \bar{X}}{\partial \bar{s}}$, $\frac{\partial \bar{X}}{\partial \bar{t}}$ and $\frac{\partial \bar{s}}{\partial s}, \frac{\partial \bar{s}}{\partial t}$ and $\frac{\partial \bar{t}}{\partial s}, \frac{\partial \bar{t}}{\partial t}$.

Problem 23 (this is partly a continuation of the previous problem) Recall the surface integral of a vector field $\vec{F}$ on a surface S parameterized by $X : D \to S$ was defined by $\int_S \vec{F} \bullet d\vec{S} = \iint_D \vec{F}(X(s,t)) \bullet \left(\frac{\partial X}{\partial s} \times \frac{\partial X}{\partial t}\right) ds\, dt$. Show that this definition is independent of the choice of parametrization. In particular, show that if you replace the expressions in terms of X and s,t in terms of $\bar{X}$ then you obtain the surface integral written in terms of the barred-parametrization. However, this is only true if we impose a certain condition on T. What condition is that?

Problem 24 Show that if $F : \mathbb{R}^n \to \mathbb{R}^n$ is invertible on $U \subseteq \mathbb{R}^n$ then dF_{x_o} is invertible for each $x_o \in U$.

Problem 25 Edwards #3.6 on page 88.

Problem 26 Edwards #3.11 on page 89.

Problem 27 Suppose $\sin w = \exp(xyz)$ and $z^3 = x^2 + y^2 + \ln w$. Find $\left.\frac{\partial w}{\partial x}\right|_y$, $\left.\frac{\partial w}{\partial x}\right|_z$ at such points as the implicit function theorem allows. Comment in each case on your choice of dependent and independent variables.

Problem 28 Edwards #3.5 on page 194. (if it is possible, follow our intuitive proof to obtain approximate the solution)

Problem 29 Edwards #3.6 on page 194.(if it is possible, follow our intuitive proof to obtain approximate the solution)

Problem 30 Edwards #3.7 on page 194.(if it is possible, follow our intuitive proof to obtain approximate the solution)