Problem 31 A complex number $a+ib$ can be represented by a 2×2 matrix $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. Let $\Psi(a+ib) = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. It is clear this mapping is a vector space isomorphism. In addition,

(a.) show that $\Psi(zw) = \Psi(z)\Psi(w)$ where zw denotes complex number multiplication and $\Psi(z)\Psi(w)$ denotes matrix multiplication,

(b.) if $||A||^2 = \text{trace}(A^TA)$ for matrices and $|a+ib|^2 = a^2+b^2$ for complex numbers then show $||\Psi(a+ib)||^2 = 2|a+ib|^2$.

Problem 32 We derived that $f = (u,v) : \mathbb{R}^2 \to \mathbb{R}^2$ is **complex differentiable** on $U \subseteq \mathbb{R}^2$ iff $f' = \begin{bmatrix} u_x & -v_x \\ v_x & u_x \end{bmatrix}$ on U. Complex differentiabiity is a special structure, take Math 331 if you'd like to know a lot more about this. Calculate the Jacobians of the following maps and determine if these real mappings denote complex-differentiable functions on $\mathbb{R}^2$:

(a.) $f(x,y) = (x^2-y^2, 2xy)$

(b.) $f(x,y) = (y,x)$

(c.) $f(x,y) = (e^x\cos(y), e^x\sin(y))$

Incidentally, the examples above in complex notation $z = x+iy$ are $f(z) = z^2$, $i\bar{z}$ and e^z.

Problem 33 Suppose the solution set of $x_1+x_2+2x_4 = 11$ and $2x_1 - x_3 = -1$ describes a plane S in $\mathbb{R}^4$. Find the equations of the plane at $(1,2,3,4)$ which is normal to S.

Problem 34 Let $\gamma(t) = (t,t^2,t^3,t^4)$ parametrize the curve C and let $\gamma(1) = p$. Find T_pC and N_pC (your choice, you can either express the tangent or normal space impicitly as a level set or expicitly via a parametrization)

Problem 35 Let $G(x,y,z,w) = (x^2+y^2+w,\ y^2+z^2-w)$. Define the affine space $S = G^{-1}(-1,10)$. Consider the point $p = (1,2,0,-6) \in S$. Calculate T_pS and N_pS and express each as follows:

(a.) $T_pS = p + \text{span}\{v_1, v_2\}$

(b.) $N_pS = p + \text{span}\{w_1, w_2\}$

Problem 36 Continuing the previous problem, Calculate T_pS and N_pS as follows:

(a.) $T_pS = F^{-1}\{(0,0)\}$

(b.) $N_pS = H^{-1}\{(0,0)\}$

where F and H are mappings from $\mathbb{R}^4$ to $\mathbb{R}^2$ whose level sets give the tangent and normal planes in $\mathbb{R}^4$ as indicated.

Problem 37 Edwards #5.4 on page 116. (Lagrange Multipliers)

Problem 38 Edwards #5.10 on page 116. (Lagrange Multipliers)

Problem 39 Edwards #5.11 on page 116. (Lagrange Multipliers)

Problem 40 Edwards #5.12 on page 116. (Lagrange Multipliers)