**Problem 41** Let $Q(x, y) = x^2 - 2xy$. Find the matrix of $Q$, what are its eigenvalues? Write the simple formula for $Q$ in terms of the eigencoordinates $\bar{x}, \bar{y}$.

**Problem 42** Let $Q(x, y) = 2xy$. Find the matrix of $Q$, what are its eigenvalues? Write the simple formula for $Q$ in terms of the eigencoordinates $\bar{x}, \bar{y}$.

**Problem 43** Let $Q(x, y, z) = 2(xy + yz)$. Find the matrix of $Q$, what are its eigenvalues? Write the simple formula for $Q$ in terms of the eigencoordinates $\bar{x}, \bar{y}, \bar{z}$.

**Problem 44** Suppose $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ where $f$ has a single local minimum at $x = x_o$ and $g$ has only one local maximum at $y = y_o$. Furthermore, suppose $f$ and $g$ are nonzero on $\mathbb{R}$. If we define $h(x, y) = [f(x)g(y)]^2$ then does $h$ have any local extrema? Find the taylor expansion of $h$ near any critical point(s) in terms of the values of $f$ and $g$ and their derivatives. Break into cases if necessary.

**Problem 45** work Edwards problem 7.10 from page 141.

**Problem 46** work Edwards problem 7.12 from page 141.

**Problem 47** Consider the function $f(x, y, z) = e^{-z^2} \sin(x^2 + y^2)$. Find points at which this function takes on local extreme values.

**Problem 48** Suppose we define $e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$. Show that $e^x e^y = e^{x+y}$ by multiplying the series via the Cauchy product. Partial credit can be obtained for working this out to third order. You may use the Binomial Theorem.

**Problem 49** The inertia tensor $[I_{ij}]$ for a rigid object $B$ provides nice formulas to describe the possible rotational motions of the rigid body. In particular, we define,

$$I_{ij} = \iiint_B \rho\big[(x_1^2 + x_2^2 + x_3^2)\delta_{ij} - x_i x_j\big]dV$$

It can be shown that if the body $B$ rotates with angular velocity $\vec{\omega}$ then the total kinetic energy of $B$ is given by

$$T = \sum_{i,j=1}^{3} I_{ij}\omega_i\omega_j = \vec{\omega}^T I\vec{\omega}.$$

The total angular momentum of $B$ is given by

$$\vec{L} = \sum_{i,j=1}^{3} I_{ij}\omega_i e_j = I\vec{\omega}.$$

Finally, the net torque on $B$ governs the change in the angular momentum:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$$

**Show that if $\vec{\tau} = 0$ then the kinetic energy is conserved. In other words, show that if $\vec{\tau} = 0$ then $\frac{dT}{dt} = 0$. warning: if your solution is longer than about a line then you're not thinking about this in the best way.**

**Problem 50** Consider the intertia tensor below:

$$[I_{ij}] = \frac{Ma^2}{12}\begin{bmatrix} 8 & -3 & -3 \\ -3 & 8 & -3 \\ -3 & -3 & 8 \end{bmatrix}$$

This is the inertia tensor for a cube of side length $a$ with one corner at the origin. The total mass $M$ of the cube is distributed evenly to give $\rho = M/a^3$. For convenience, assume $\frac{Ma^2}{12} = 1$ and find the eigenvalues and eigenvectors. Determine the physical significance of the eigenvectors as the relate to the symmetries of the cube.