Same instructions as Mission 1. Thanks!

Problem 25 Consider $\gamma(t) = (t, t^2/2, 4, -t)$ for $t \in \mathbb{R}$. Let $C = \gamma(\mathbb{R})$. Find the tangent and normal space to C at $\gamma(2)$.

Problem 26 Consider $F(x, y, z, w) = (x^2 + y^2, z^2 - w^2)$. Define $M = F^{-1}(5, -7)$.

(a.) Find the tangent space and normal space to M at the point $p = (1, 2, 3, 4)$.

(b.) Find a parametrization of M near $p = (1, 2, 3, 4)$ and find T_pM via a calculation involving the parametrization

Problem 27 Consider $F(x, y, z, t) = x^2 + y^2 + z^2 - t^2$. Let $M = F^{-1}(0)$ and $p = (\ 1, \sqrt{2}, \sqrt{3}, \sqrt{6}\ )$.

(a.) Find the normal space to M at p,

(b.) Find a parametrization of M and use it to calculate T_pM.

Problem 28 Let $S_R(x_o, y_o)$ be the circle of radius R centered at (x_o, y_o).

(a.) Find a parametrization of $M = S_R(x_o, y_o) \times S_A(x_1, y_1) \subseteq \mathbb{R}^4$. Find the tangent space at an arbitrary point in M

(b.) Express $M = S_R(x_o, y_o) \times S_A(x_1, y_1) \subseteq \mathbb{R}^4$ as the level-set of an appropriate function. Find the normal space to M at an arbitrary point on M.

Problem 29 Use the method of Lagrange multipliers to find the distance between the unit-circle $x^2 + y^2 = 1$ and the line $x + y = 4$

Problem 30 Find the highest and lowest points on the ellipse of intersection of the cylinder $x^2+y^2 = 1$ and the plane $x + y + z = 1$.

Problem 31 Use the method of Lagrange multipliers to find the minimum distance from the origin to the curve of intersection of the surfaces $z^2 = x^2 + y^2$ and $x - 2z = 3$.

Problem 32 Let A be a symmetric matrix; $A^T = A$. Define $Q(x) = x^T Ax$ for each $x \in \mathbb{R}^n$. Apply the method of Lagrange multipliers to find the condition for min/max of Q restricted to $S_{n-1} = \{x \in \mathbb{R}^n \mid \|x\| = 1\}$ (here I use $\|x\|^2 = x^T x$, that is $\|x\|$ is the Euclidean norm).

Bonus 4: Suppose we have two manifolds in $\mathbb{R}^n$ given as level sets of

$$g : \mathbb{R}^n \to \mathbb{R}^m \qquad \& \qquad h : \mathbb{R}^n \to \mathbb{R}^k.$$

Let $M = g^{-1}(0)$ and $N = h^{-1}(0)$ such that $M \cap N = \emptyset$. If there exist $p \in M$ and $q \in N$ which are closest together of all pairs of points $(p, q) \in M \times N$ then prove the line-segment $[p, q] = \{p+t(q-p) \mid 0 \leq t \leq 1\}$ is orthgonal to both M at p and N at q.
Please use the method of Lagrange multipliers to minimize $f(x, y) = \|x\|^2 + \|y\|^2$ for $x, y \in \mathbb{R}^n$ subject $G(x, y) = (g(x), h(y)) = (0, 0)$.