Same instructions as Mission 1. Thanks!

**Problem 57** Consider $F : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $F(r,\theta,\phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$. View $r,\theta,\phi$ as Cartesian coordinates on the domain of $F$ and view $x,y,z$ as coordinates for the codomain of $F$. In particular, this indicates $F^1 = x$ and $F^2 = y$ and $F^3 = z$. Find $F^*(\beta)$ where $\beta = \frac{1}{x^2+y^2}[-ydx + xdy]$.

**Problem 58** (Lee p.389 Problem 11) Let $\omega$ below defined for $(x,y,z) \neq 0$,

$$\omega = \frac{x\,dy \wedge dz + y\,dz \wedge dx + z\,dx \wedge dy}{(x^2+y^2+z^2)^{3/2}}$$

Calculate $d\omega$. Is $\omega$ closed? Is $\omega$ exact? Express $\omega$ in spherical coordinates (pull-back $dx, dy, dz$ etc. to $d\rho, d\phi, d\theta$ etc.)

**Problem 59** Renteln Exercises 3.32 and 3.33 page 97. (pull-backs)

**Problem 60** Renteln Exercise 3.35 page 98. (relation of push-forward to Jacobian matrix )

**Problem 61** Let $U$ be open subset of $\mathbb{R}^n$. Suppose $\omega_1$ is a closed differential form on $U \subseteq \mathbb{R}^n$ and suppose $\omega_2$ is an exact differential form on $U \subseteq \mathbb{R}^n$. Show $\omega_1 \wedge \omega_2$ is exact.

**Problem 62** Renteln Exercise 3.43 page 105. ( Lie groups )

**Problem 63** Renteln Exercise 3.44 page 106. ( Lie algebras )

**Problem 64** Renteln Exercise 3.46 page 108. ( calculate a Lie algebra )

**Bonus 8:** Renteln Exercise 3.57 page 113-114. ( symplectic forms and Hamiltonians )