Same instructions as Mission 1. Thanks!

Problem 37 Your signature below indicates you have:

(a.) I have read much of Cook's Chapter 4 and 5:________________.

Problem 38 Let $F(x, y) = (x^3 - 3xy^2, 3x^2 - y^3)$ for all $(x, y) \in \mathbb{R}^2$. Show F is locally invertible at all points in the plane except one. Find the inverse for F restricted to the sector $-\pi/3 < \theta < \pi/3$ for $r > 0$ (I use the usual polar coordinates in the plane)

Problem 39 Let $F(x, y, z) = \dfrac{1}{x^3 + y^3 + z^3 - 3xyz}(x^2 - yz, z^2 - xy, y^2 - xz)$. Find the inverse function of F, or, if not globally possible, find a local inverse for F.

Problem 40 Edwards, page 194, #3.7.

Problem 41 Let $G(x, y, a, b) = (x^2 - y^2 - ax + by, 2xy - xb - ya)$. Suppose $M = G^{-1}(2, 1)$.

(a.) Solve for a, b as functions of x, y

(b.) use the implicit function theorem to show where it is possible to solve for a, x as functions of b, y (no need to actually solve it, demonstration of existence suffices)

(c.) use the implicit function theorem to show where it is possible to solve for a, y as functions of b, x (no need to actually solve it, demonstration of existence suffices))

(d.) use the implicit function theorem to show where it is possible to solve for x, y as functions of a, b. (no need to actually solve it, demonstration of existence suffices))

note: I don't expect you to analyze the subtle question of if it is still possible to solve where there implicit function theorem breaks down. I merely wish for you to find the low-hanging fruit which the implicit function theorem provides

Problem 42 Consider $F(x, y, z, w) = (x^2 + y^2, z^2 - w^2)$. Define $M = F^{-1}(5, -7)$. Find the tangent space and normal space to M at the point $p = (1, 2, 3, 4)$.

Problem 43 Find a parametrization of M in the previous problem near the given point. Verify the tangent space you found by utilizing the parametrization of M as appropriate.

Problem 44 Let $G(x, y, z, w) = x^2 + y^2$ and consider $M = G^{-1}(2)$. Find the tangent space and normal space to M at the point $p = (1, 1, 2, 3)$.

Problem 45 Suppose $G_1 : \mathbb{R}^5 \to \mathbb{R}$ and $G_2 : \mathbb{R}^5 \to \mathbb{R}^2$ are smooth functions and $M_1 = G_1^{-1}\{c_1\}$ and $M_2 = G_2^{-1}\{(c_2, c_3)\}$ are non-empty manifolds where G_1 and G_2 are both full-rank; $G_1'(p)$ has rank 1 at each point in M_1 and $G_2'(q)$ has rank 2 at each point q in M_2. If we define[1] $G = (G_1, G_2)$ then explain how $M = G^{-1}\{(c_1, c_2, c_3)\}$ relates to M_1 and M_2.

[1] I make the identification $(a, (b, c)) = (a, b, c)$ here

Problem 46 Let S_1 be the unit-circle and S_2 be the unit-sphere. Find a parametrization of $M = S_1 \times S_2 \subseteq \mathbb{R}^5$. Find the tangent space at an arbitrary point in M

Problem 47 Consider M in the previous problem. Express M as the level-set of an appropriate function. Find the normal space to M at an arbitrary point on M.

Problem 48 If $x^2 + y^2 + z^2 + w^2 = 1$ and $xywz = 1$ then calculate $\frac{\partial z}{\partial x}\big|_y$. That is, take z, w to be dependent variables and calculate the derivative of z with respect to x while holding y-fixed.

Problem 49 Consider $PV = nRT$ where P, V, n, T are variables. If $P = V^2$ then calculate $\frac{\partial T}{\partial V}$ assuming $T = T(V, P)$.

Problem 50 Edwards page 116, #5.6 (Lagrange multipliers)

Problem 51 Edwards page 116, #5.9 (Lagrange multipliers)

Problem 52 Edwards page 116, #5.10 (Lagrange multipliers)

Problem 53 $\mathbb{H} = \mathbb{R} \oplus i\mathbb{R} \oplus j\mathbb{R} \oplus k\mathbb{R}$ denotes Hamilton's **quaternions**. In particular, a quaternion has the form $\alpha = t + xi + yj + zk$ for $t, x, y, z \in \mathbb{R}$ and the **versors** i, j, k satisfy the relations: $-ji = ij = k$ and $-kj = jk = i$ and $-ik = ki = j$ and $i^2 = j^2 = k^2 = -1$. Define the quaternionic conjugate of α by:

$$\bar{\alpha} = t - xi - yj - zk$$

calculate the formula for $\alpha\bar{\alpha}$ and use the given notation to define S_3 as a subset of the quaternions. (we discussed S_3 in Lecture, it is the unit 3-sphere which lives in 4-dimensional space)

Problem 54 Show that T_pM is indeed a subspace of $T_p\mathbb{R}^n$ for the case of a parametrized M

(*in my notes, I prove the implicitly defined case, but, you might notice proof that the set of all tangent vectors to a parametrized manifold is absent from my current notes. To prove this, you need to show that the sum and scalar multiple of tangent vectors are once again tangent vectors where the primary definition of a tangent vector is given in terms of curves*)