Same instructions as Mission 1. Thanks!

Problem 55 Your signature below indicates you have:

(a.) I have read much of Cook's Chapter 6 and 8: ________________.

Problem 56 Let $Q(x,y) = 10x^2 - 8xy + 10y^2$. Find a linear change of coordinates for which $Q(\bar{x},\bar{y}) = \lambda_1\bar{x}^2 + \lambda_2\bar{y}^2$.

Problem 57 Let $Q(x,y,z) = 31x^2 + 15y^2 + 15z^2 - 22xy - 22xz + 10yz$. Find the matrix for Q and use technology to calculate the eigenvalues for Q. Let y_1, y_2, y_3 denote the eigencoordinates, write the formula for Q in terms of the eigencoordinates. For the record, I did not design this problem to have that eigenvalue. It just happened.

Problem 58 Let $Q(x,y,z) = 6x^2 - 2xy - 4xz + 5y^2 - 2yz + 6z^2$. Find the matrix for Q and use technology to calculate the eigenvalues for Q. Let y_1, y_2, y_3 denote the eigencoordinates, write the formula for Q in terms of the eigencoordinates.

Problem 59 I'll share a little backwards linear algebra with you: if v_1, v_2, v_3 are orthonormal vectors then we can build a symmetric matrix by $A = \lambda_1 v_1 v_1^T + \lambda_2 v_2 v_2^T + \lambda_3 v_3 v_3^T$. Let,

$$v_1 = \frac{1}{\sqrt{3}}(1,1,1), \quad v_2 = \frac{1}{\sqrt{6}}(1,-2,1), \quad v_3 = \frac{1}{\sqrt{2}}(1,0,-1).$$

and $\lambda_1 = 3, \lambda_2 = 6$ and $\lambda_3 = 8$. **Calculate A and verify $Av_1 = 3v_1$ and $Av_2 = 6v_2$ and** $Av_3 = 8v_3$.

Problem 60 Consider $f(x,y) = 3 + 3(x-1) + 4(y+2) + 2(x-1)^2 + 6(x-1)(y+2) + 7(y+2)^2 + \cdots$. Find the values of $f, f_x, f_y, f_{xx}, f_{xy}, f_{yy}$ at the point $(1,-2)$.

Problem 61 Calculate the multivariate Taylor series centered about $(0,0,0)$ for $f(x,y,z) = \dfrac{1+y^2}{1-2xz}$ to order 4. Analyze: is $(0,0,0)$ a critical point? If so, analyze if it yields a min/max/saddle or if the second derivative test is not applicable for the given problem.

Problem 62 Find and classify the critical points of $f(x,y) = (x^2+y^2)e^{x^2-y^2}$.

Problem 63 Consider $\int \cos(tx)dx = \frac{1}{t}\sin(tx) + C$. Use differentiation with respect to t to derive a new integral or two from the given integral. Set $t = 1$ or some other fun value once you've completed the differentiation.

Problem 64 Follow §4 of Professor Conrad's blurb to derive $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$

Problem 65 Read §9 of Professor Conrad's blurb and summarize his argument in your own words.

Problem 66 Find the geodesics on the cone $\phi = \pi/3$ where ϕ denotes the usual spherical angle.

Problem 67 Find a differential equation whose solutions are geodesics on the surface $x^2 + y^3 + z^4 = 1$. **Bonus: numerically generate a particular solution and plot it on the surface using some C.A.S.**

Problem 68 Let $L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{k}{2}(x^2 + y^2)$ denote the Lagrangian of a particle with mass m under the force a spring with potential energy $U(x, y) = \frac{k}{2}(x^2 + y^2)$. Notice $L = T - U$ where T is the kinetic energy. **Calculate the Euler-Lagrange equations and show energy $E = T + U$ is conserved along the solution to the Euler-Lagrange equation**

Problem 69 Show that the geodesics to a sphere are the great circles.

Problem 70 Let $L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + g(r)$ where g is a differentiable function of the polar radius r. Find the Euler Lagrange equations. Also, suppose we define angular momentum $J = \dfrac{\partial L}{\partial \dot{\theta}}$, show J is conserved.

Problem 71 A marble slides without friction on a bowl of radius R. If the marble has mass m and the force of gravity is given by $-mg\,\widehat{z}$ then find the equations of motion for the marble (differential equations suffice as an answer here).

Problem 72 Imagine a pendulum of length l_1 which consists of a very light rod which does not flex and a bob of mass m_1. Next, a second pendulum of length l_2 which consists of a very light rod which does not flex and a bob of mass m_2 is attached so that l_2 hangs freely off m_1. All of this is attached to point and allowed to swing back and forth under the influence of gravity. Assume this is near the surface of the earth where $F = mg$ applies. Find the equations of motion for this double pendulum. Let θ_1 and θ_2 be the angles which l_2 and l_2 make with respect to $-\widehat{z}$. Write the equations of motion in terms of these anglular variables.