Show your work and justify steps.

Problem 17 [10pts] Let V, W be finite dimensional vector spaces with norms $||\cdot||_V$ and $||\cdot||_W$ respectively. Suppose $T : V \to W$ is a linear transformation. Show T is continuous.

Problem 18 [10pts] Suppose $R \in \mathbb{R}$ is a fixed, positive constant. Let $X : \mathbb{R}^3 \to \mathbb{R}^4$ be defined by

$$X(\theta, \phi, \psi) = (R\cos\theta\sin\phi\sin\psi, R\sin\theta\sin\phi\sin\psi, R\cos\phi\sin\psi, R\cos\psi).$$

Let $X(\mathbb{R}^3) = V$. Let $F(x, y, z, t) = x^2 + y^2 + z^2 + t^2$. Show that $V = F^{-1}\{R\}$. Let $p = X(\pi/4, \pi/4, \pi/6)$. Find T_pV and N_pV. You may describe T_pC and N_pC as a span or as point-sets in $\mathbb{R}^4$ given by cartesian equations, your choice.

Problem 19 [10pts] Suppose we wish to find the extrema of $F : \mathbb{R}^n \to \mathbb{R}$ on some compact domain given by $G^{-1}\{0\}$ where $G = (G_1, \dots, G_p) : \mathbb{R}^n \to \mathbb{R}^p$. Consider the function $H(x, \lambda_1, \dots, \lambda_p) = F(x) - \sum_{i=1}^{p} \lambda_i G_i(x)$ where $H : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$. Explain what critical points of H yield.