

Quiz 10

$$1.) \mathcal{L} \{ t e^{-2t} + 3 \} (s) = \boxed{\frac{1}{(s+2)^2} + \frac{3}{s}}$$

$$2.) \mathcal{L} \{ t e^{-2t} \sin(3t) \} (s) = -\frac{d}{ds} \left[\frac{3}{(s+2)^2 + 9} \right] = \boxed{\frac{6(s+2)}{[(s+2)^2 + 9]^2}}$$

$$3.) \mathcal{L} \{ e^t u(t-3) \} (s) = e^{-3s} \mathcal{L} \{ e^{t+3} \} (s) = e^3 e^{-3s} \mathcal{L} \{ e^t \} (s) \\ = \boxed{\frac{e^{3(1-s)}}{s-1}}$$

$$4.) F(s) = \frac{6s}{s^2+4s+13} = \frac{6(s+2) - 12}{(s+2)^2 + 9} = \frac{6(s+2) - 4(3)}{(s+2)^2 + 9}$$

$$\Rightarrow \mathcal{L}^{-1} \{ F \} (t) = \boxed{6e^{-2t} \cos 3t - 4e^{-2t} \sin 3t}$$

$$5.) F(s) = \frac{1}{s(s-1)} e^{-2s}$$

$$G(s) = \frac{A}{s} + \frac{B}{s-1} \Rightarrow A(s-1) + Bs = 1$$

$$\underline{s=1} \quad B=1$$

$$\underline{s=0} \quad -A=1$$

$$F(s) = \left(\frac{1}{s-1} - \frac{1}{s} \right) e^{-2s}$$

$$\mathcal{L}^{-1} \{ G(s) e^{-2s} \} (t) = \mathcal{L}^{-1} \{ G(s) \} (t-2) u(t-2)$$

$$= (e^{\bar{t}} - 1) \Big|_{\bar{t}=t-2} u(t-2)$$

$$= \boxed{(e^{t-2} - 1) u(t-2)}$$