MATH 334: SPRING 2012:	QUIZ 1

Please put your work on this page. Box your answers. Thanks and enjoy. Problems 1 and 2 are basic problems which ought not involve a substitution. However, Problem 3 is a Bernoulli equation as we discussed in lecture last week. You have 10 minutes to complete this quiz.

Problem 1 Find the general solution of $\frac{dy}{dx} = xy$.

$$\frac{dy}{y} = x\,dx \quad \Rightarrow \quad \ln|y| = \frac{1}{2}x^2 + C$$

$$\boxed{y = ke^{\frac{1}{2}x^2}}$$

Problem 2 Find the general solution of $2xy^2dx + (2x^2y - \sin(y))dy = 0$.

Let $F(x,y) = x^2y^2 + \cos(y)$

note $\frac{\partial F}{\partial x} = 2xy^2$ & $\frac{\partial F}{\partial y} = 2x^2y - \sin(y)$

$\therefore$ $\boxed{x^2y^2 + \cos(y) = C}$ is sol'n

Problem 3 Solve $\frac{dy}{dt} + \frac{1}{t}y = -ty^3$. $\Rightarrow$ $y^{-3}\frac{dy}{dt} + \frac{1}{t}y^{-2} = -t$

Let $z = y^{-2}$ $\quad \frac{dz}{dt} = -2y^{-3}\frac{dy}{dt}$ $\quad \therefore$ $y^{-3}\frac{dy}{dt} = \frac{-1}{2}\frac{dz}{dt}$

$\therefore$ $\frac{-1}{2}\frac{dz}{dt} + \frac{1}{t}z = -t$

$\frac{dz}{dt} - \frac{2}{t}z = 2t$ $\Rightarrow$ $I = \exp\left(\int \frac{-2\,dt}{t}\right) = e^{-2\ln|t|} = e^{\ln|t|^{-2}} = \frac{1}{t^2}$

$\frac{1}{t^2}\frac{dz}{dt} - \frac{2}{t^3}z = \frac{2}{t}$

$\frac{d}{dt}\left(\frac{1}{t^2}z\right) = \frac{2}{t}$ $\quad \frac{1}{t^2}z = 2\ln|t| + C$

$\boxed{y^{-2} = 2t^2\ln(t) + Ct^2}$ $\therefore$ $\left(y = \frac{\pm 1}{\sqrt{2t^2\ln t + Ct^2}}\right)$