

Please put your work on this page. Box your answers. Thanks and enjoy. You have 12 minutes to complete this quiz.

Problem 1 Solve the following differential equations.

a. $(D^2 + 6D + 40)[y] = 0$

$$[(D+3)^2 + 31][y] = 0 \quad \hookrightarrow \quad \underline{y = C_1 e^{-2x} \cos \sqrt{31} x + C_2 e^{-2x} \sin \sqrt{31} x}$$

b. $(D^4 + 5D^2 - 6)[y] = 0$

$$(D^2 + 6)(D^2 - 1)[y] = (D^2 + 6)(D+1)(D-1)[y] = 0$$

$$\hookrightarrow \quad \underline{y = C_1 \cos \sqrt{6} x + C_2 \sin \sqrt{6} x + C_3 e^x + C_4 e^{-x}}$$

c. $(D-1)^2(D^2+1)^2[y] = 0$

$$\underline{y = C_1 e^x + C_2 x e^x + C_3 \cos x + C_4 \sin x + C_5 x \cos x + C_6 x \sin x}$$

Problem 2 Let $T = x^2 D^2 + D$. Calculate $T[xe^x]$.

$$\begin{aligned} T[xe^x] &= (x^2 D^2 + D)[xe^x] \\ &= x^2 D^2 [xe^x] + D[xe^x] \\ &= x^2 (x'' e^x + 2x'(e^x)' + x(e^x)'') + e^x + xe^x \\ &= x^2 (2e^x + xe^x) + e^x + xe^x = \underline{e^x (2x^2 + x^3 + 1 + x)} \end{aligned}$$

Problem 3 Find an annihilator for

a. $f_1(x) = x^2$

$$\lambda = 0 \text{ thrice } \hookrightarrow \underline{A_1 = D^3}$$

b. $f_2(x) = \cos(2x)$

$$\lambda = \pm 2i \quad \hookrightarrow \underline{A_2 = D^2 + 4}$$

Problem 4 Set-up the particular solution for $y'' + 4y = g(x)$ in the cases (do not find A, B, ... etc..)

a. $g(x) = f_1(x) = x^2$

$$(D^2 + 4)[y] = g$$

$$D^3 (D^2 + 4)[y] = D^3 [x^2] = 0 \quad \hookrightarrow \quad \underline{y_p = Ax^2 + Bx + C}$$

b. $g(x) = f_2(x) = \cos(2x)$

$$(D^2 + 4)(D^2 + 4)[y] = (D^2 + 4)[\cos 2x] = 0$$

$$y = C_1 \cos 2x + C_2 \sin 2x + \underbrace{C_3 x \cos 2x + C_4 x \sin 2x}$$

$$\hookrightarrow \quad \underline{y_p = Ax \cos 2x + Bx \sin 2x}$$