Please put your work on this page. Box your answers. Thanks and enjoy. You have 16 minutes to complete this quiz. Don't forget the bonus problem on the back, it's worth 10pts.

Problem 1 [2pts] Find an annihilator for

(a.) $f_1(x) = e^{-3x} + x^2$

$\lambda = -3$ and $\lambda = 0$ thrice $\Rightarrow$ $A = (D+3)D^3$

(b.) $f_2(x) = xe^x \cos(2x)$

$\lambda = 1 \pm 2i$ twice $\Rightarrow$ $A = \left((D-1)^2 + 4\right)^2 = (D^2 - 2D + 5)^2$

Problem 2 [2pts] Set-up, but do not find $A, B, C, ...$, the particular solution for:

(a.) $y'' - y = x^2$

$(D^2-1)[y] = (D+1)(D-1)[y] = x^2 \Rightarrow D^3(D+1)(D-1)[y] = D^3[x^2] = 0.$

$\therefore y_p = Ax^2 + Bx + C$.

(b.) $(D^2+9)[y] = \cos(3\theta) + e^\theta$

$(D-1)(D^2+9)(D^2+9)[y] = (D-1)(D^2+9)[\cos 3\theta + e^\theta] = 0$

(overlap) $\Rightarrow$ $y_p = Ae^\theta + B\theta\sin\theta + C\theta\cos\theta$.

Problem 3 [6pts] Solve $y'' - 4y' = e^x + 3$

$(D-1)D(D^2-4D)[y] = D(D-1)[e^x+3] = 0.$

$(D-1)D^2(D-4)[y] = 0$

$y_h = c_1 + c_2 e^{4x}$ and $y_p = Ae^x + Bx$

$y_p' = Ae^x + B$

$y_p'' = Ae^x + 0$

$y_p'' - 4y_p' = e^x + 3$

$Ae^x - 4(Ae^x + B) = e^x + 3$

$-3Ae^x - 4B = e^x + 3$

$-3A = 1$ AND $-4B = 3$

$A = -1/3$ and $B = -3/4$

$$y = c_1 + c_2 e^{4x} - \frac{1}{3}e^x - \frac{3}{4}x$$

Problem 4 [10pts, bonus!] Solve, for $x > 0$,

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

Let $y = x^R$ note $y' = Rx^{R-1}$ and $y'' = R(R-1)y$

Substituti,

$$2x^2R(R-1)x^{R-2} - RxX^{R-1} + x^R = 0$$

$$(2R(R-1) - R + 1)x^R = 0 \quad \text{for } x > 0$$

$$\Rightarrow 2R^2 - 2R - R + 1 = 0 \quad \text{(characteristic eq}^n\text{)}$$

$$2R^2 - 3R + 1 = 0$$

$$(2R - 1)(R - 1) = 0$$

$$R_1 = 1/2 \quad \text{and} \quad R_2 = 1$$

$$\boxed{y = C_1\sqrt{x} + C_2x}$$